Bichromatic squeezed light from non-degenerate optical parametric oscillators

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Quantum Light

In the quantum description of light one would expect energy and phase to form a canonical pair

$$\left[\hat{N},\hat{\phi}\right]=i\hbar$$

but this is not so... because a phase operator proper does not exist!

Maybe not that weird: phases cannot be measured, only phase differences can. The phase difference operator exists (Luis & Sánchez Soto) and behaves well.

Hence in describing phase properties of quantum light a different strategy must be followed: quadratures.

The monochromatic classical wave

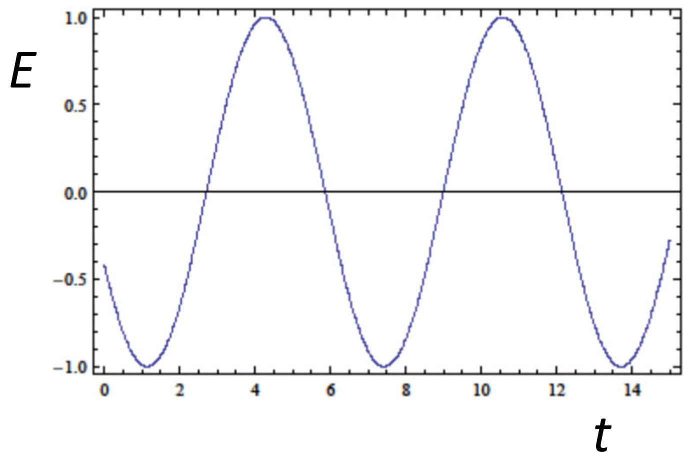
$E(t) = E_0 \cos(\omega t + \phi_0)$

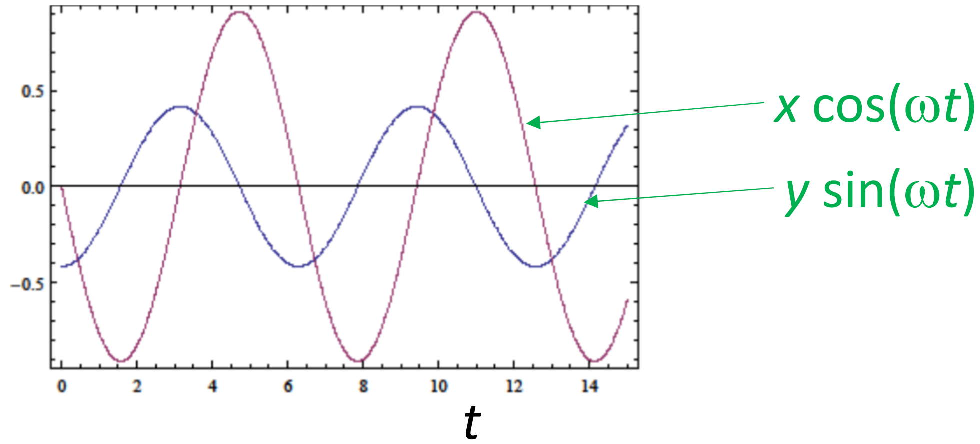
can be rewritten as

$E(t) = E_0 \cos(\omega t + \phi_0) = x \sin(\omega t) + y \cos(\omega t)$

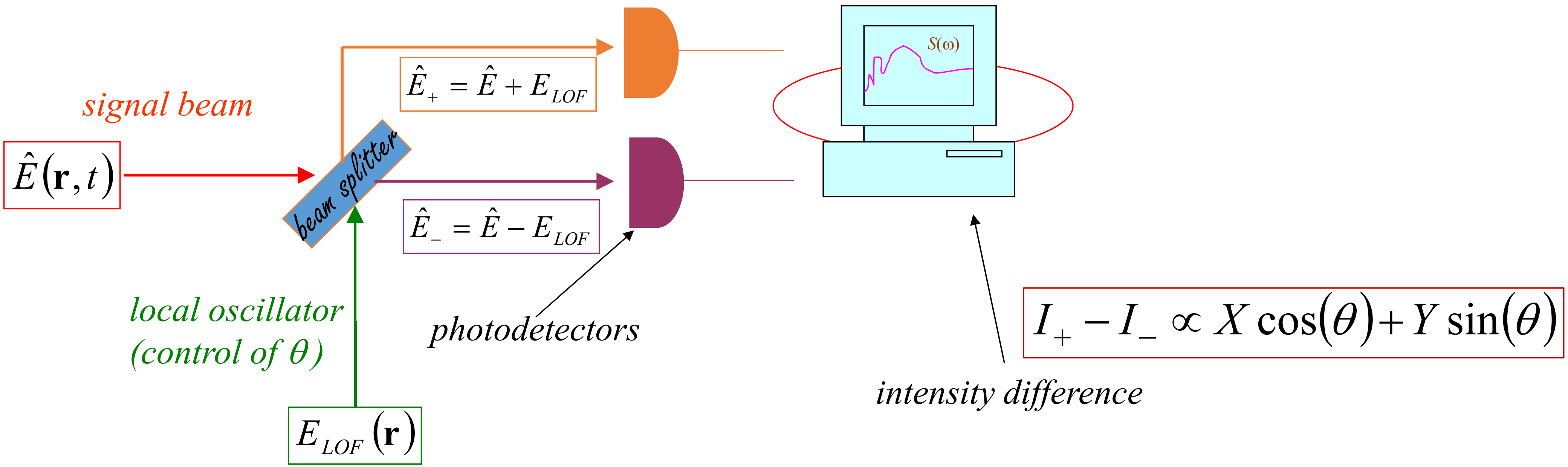
with $\begin{cases} x = E_0 \sin\phi_0 \\ y = E_0 \cos\phi_0 \end{cases}$

and quadratures can be measured





Balanced homodyne detection



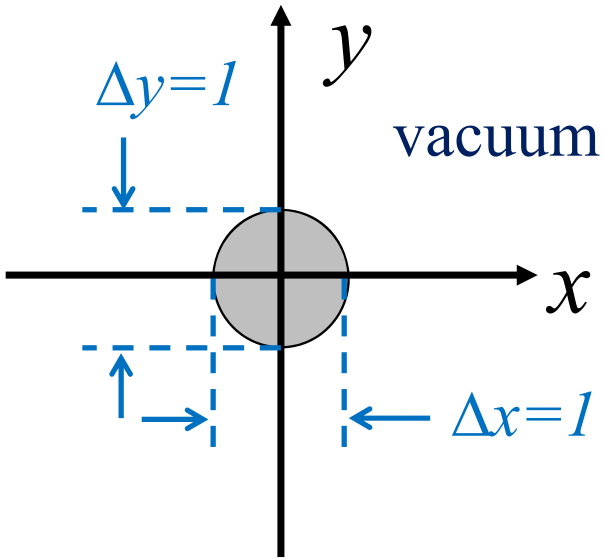
$$\widehat{\vec{E}}(\vec{r},t) = i\vec{e}\mathcal{E}\left(\hat{a}e^{-i\Phi} - \hat{a}^\dagger e^{i\Phi}\right) = \vec{e}\mathcal{E}\left(\hat{x}\sin\Phi - \hat{y}\cos\Phi\right)$$

$$\hat{x} \equiv \hat{a}^\dagger + \hat{a}, \quad \hat{y} \equiv i\left(\hat{a}^\dagger - \hat{a}\right) \qquad \Delta\hat{x}\Delta\hat{y} \geqslant 1.$$

$$\hat{x}^\theta = e^{-i\theta}\hat{a}^\dagger + e^{i\theta}\hat{a}$$

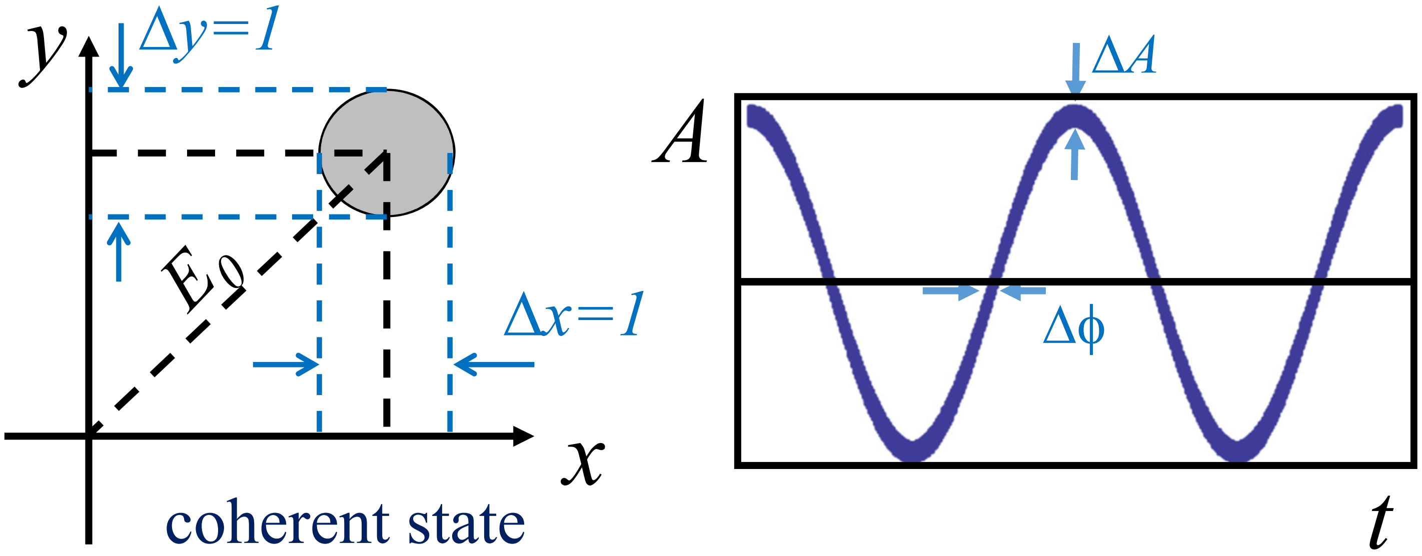
$\langle x \rangle, \quad \langle x^2 \rangle$

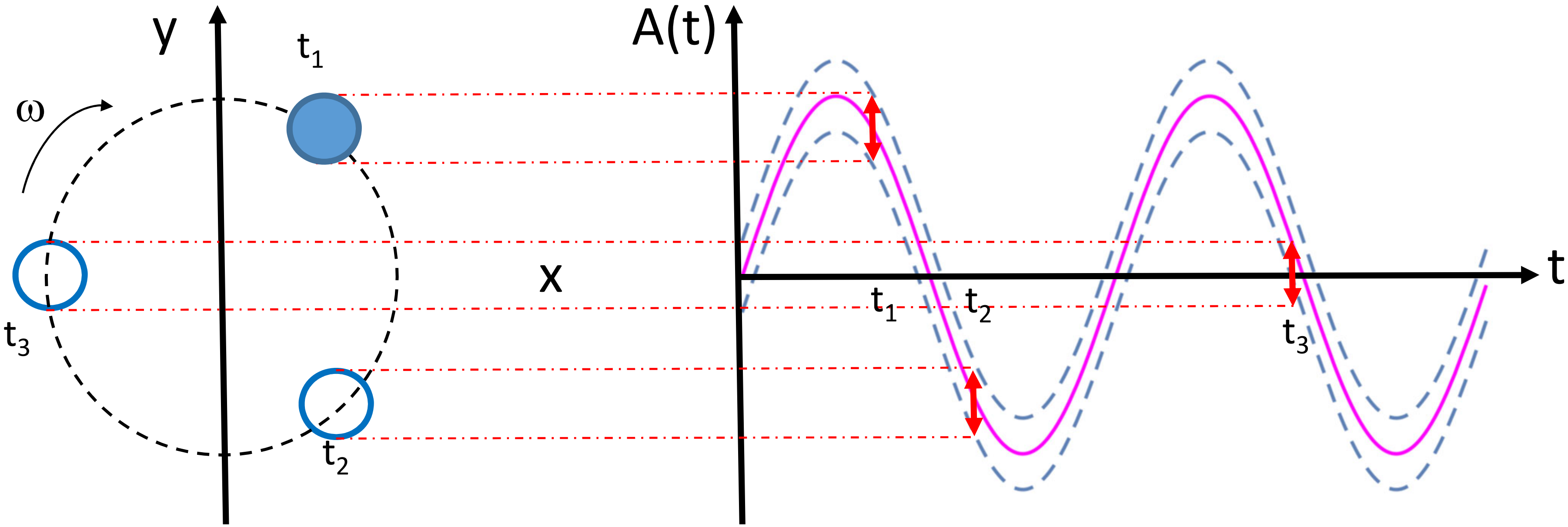
$\Delta x^2 = \langle x^2 \rangle - \langle x \rangle^2$



$\Delta x \Delta y = 1$

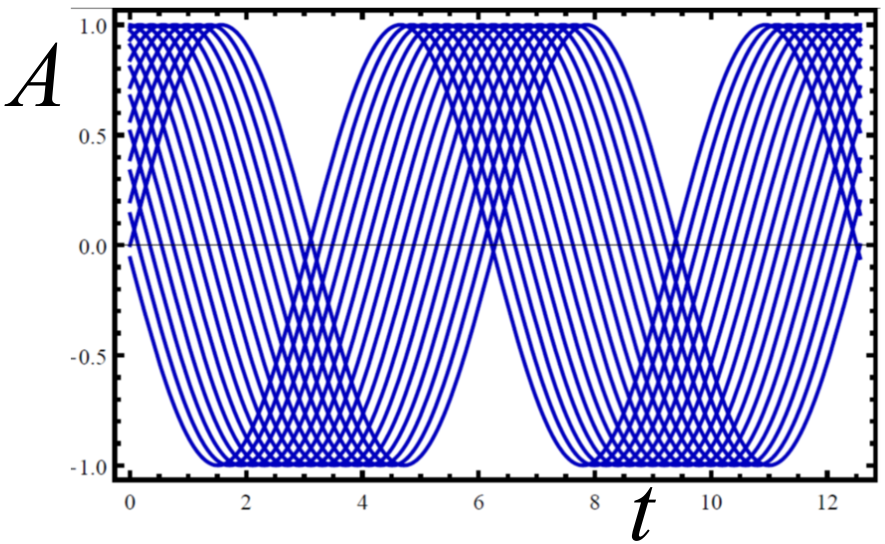
$$A(t) = x \sin(\omega t) + y \cos(\omega t)$$

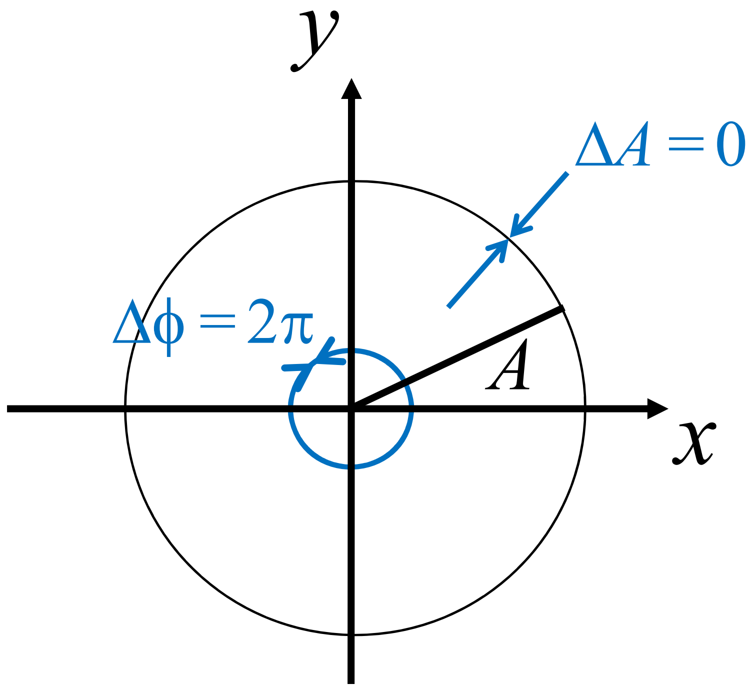




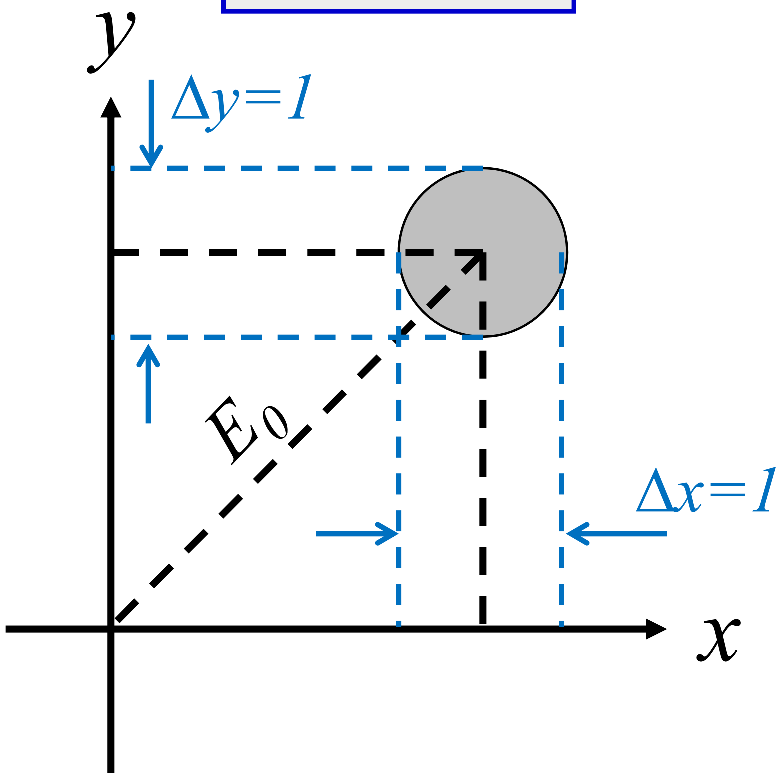
y
t1
ω
t3
t2
x
A(t)
t
t1
t2
t3

How does a Fock state look like in this representation?

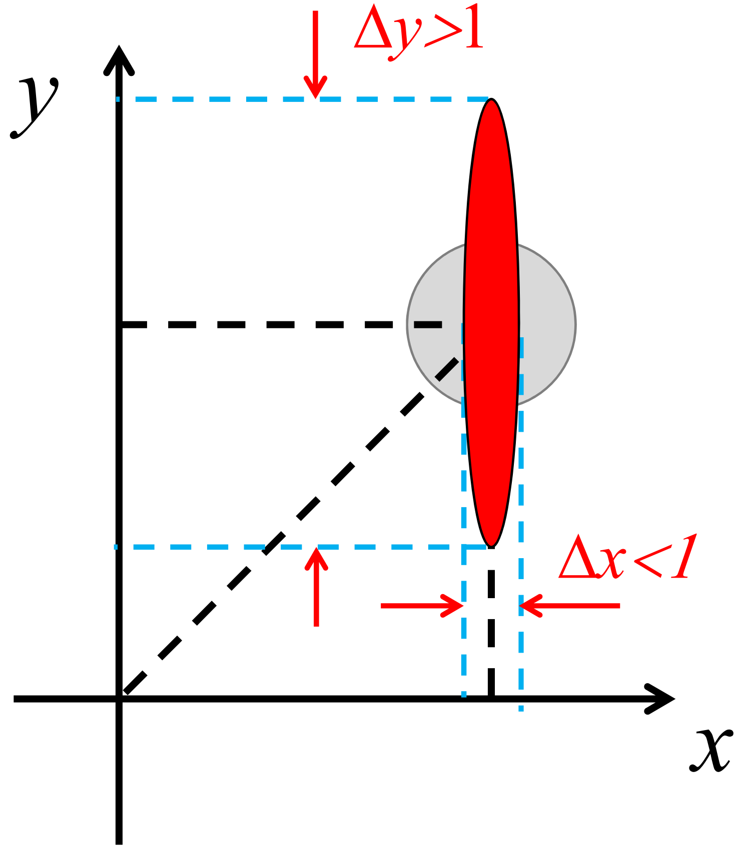




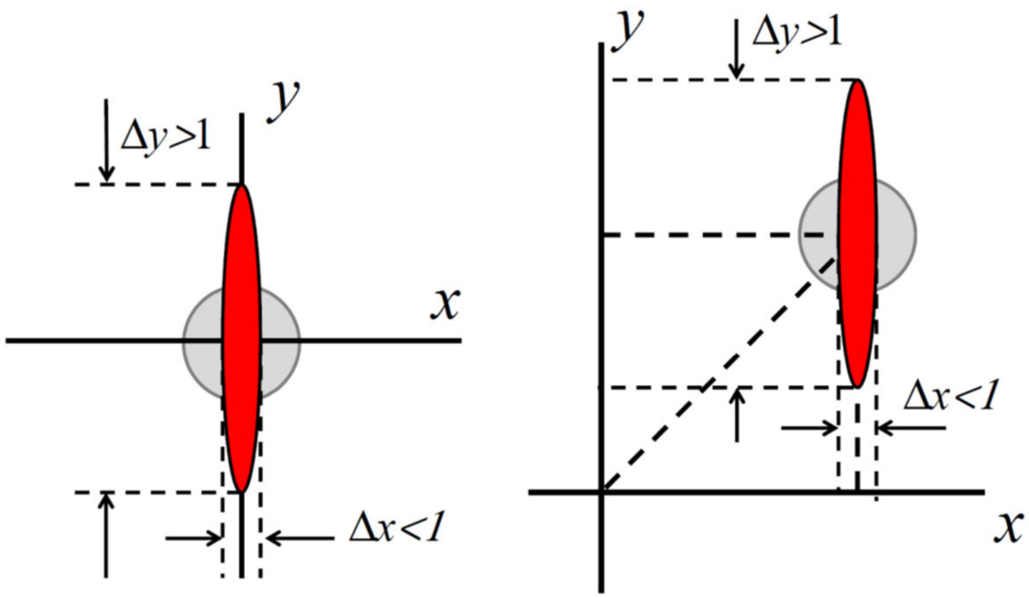
Squeezing



coherent state

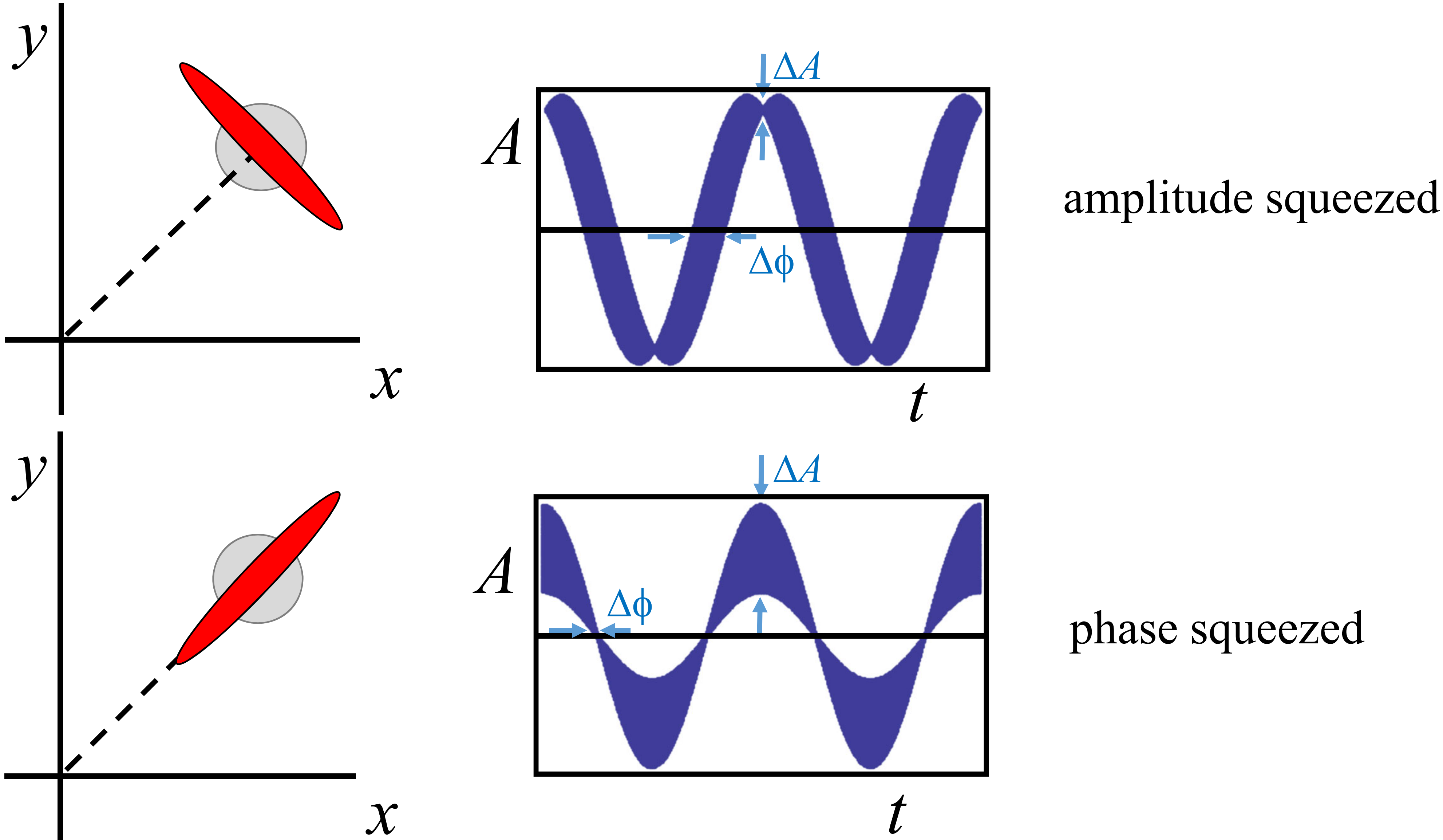


squeezed state



(a) Vacío comprimido

(b) estado comprimido "brillante"



y
x
A
t
ΔA
Δϕ
amplitude squeezed
y
x
A
t
ΔA
Δϕ
phase squeezed

How could this be done?

One needs a unitary transformation that must be non-linear in the bosonic operators

$$\hat{S}(z) = \exp\left[\frac{1}{2}\left(z^*\hat{a}^2 - z\hat{a}^{\dagger 2}\right)\right]$$

$$|sq\ vac\rangle = \hat{S}(z)|0\rangle$$

Se define el **estado coherente a dos fotones**, como

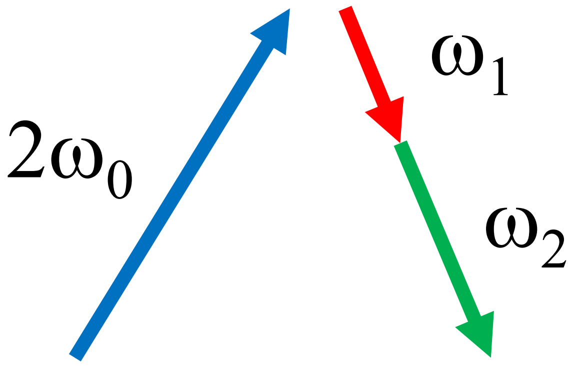
$$|\beta\rangle_z = |z;\alpha\rangle \equiv \hat{S}(z)|\alpha\rangle = \hat{S}(z)\hat{D}(\alpha)|0\rangle$$

se define el **estado comprimido ideal**, como

$$|\alpha;z\rangle \equiv \hat{D}(\alpha)\hat{S}(z)|0\rangle\ ,$$

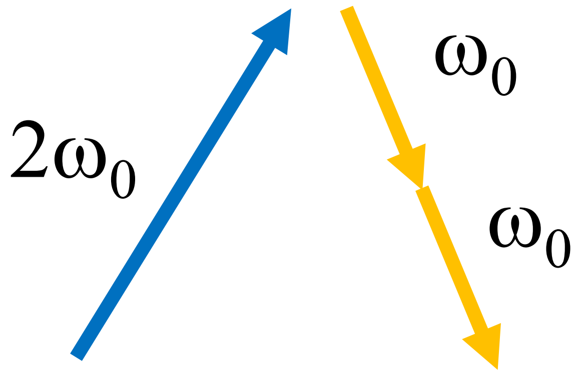
Optical Parametric Oscillators

Parametric down conversion comes in two different forms



Non-degenerate

$$\hat{S}_2(\xi) = \exp\left(\xi^* \hat{a}\hat{b} - \xi \hat{a}^\dagger \hat{b}^\dagger\right),$$

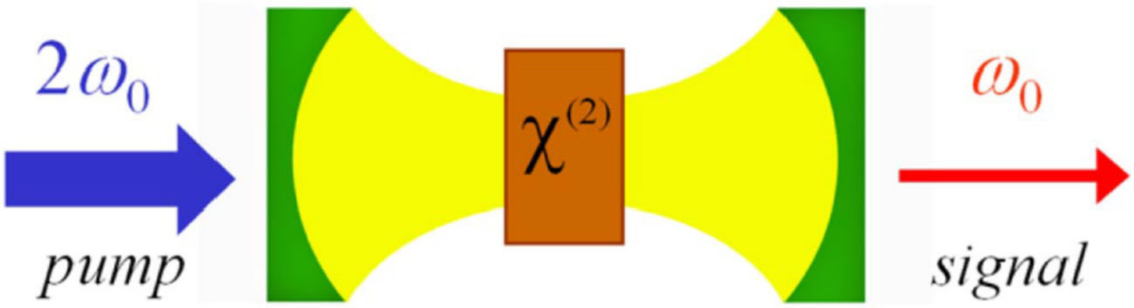


degenerate

$$\hat{S}(z) = \exp\left[\frac{1}{2}\left(z^* \hat{a}^2 - z \hat{a}^{\dagger 2}\right)\right]$$

and when these processes occur intracavity, they give rise to two different types of optical parametric oscillators …

DOPO



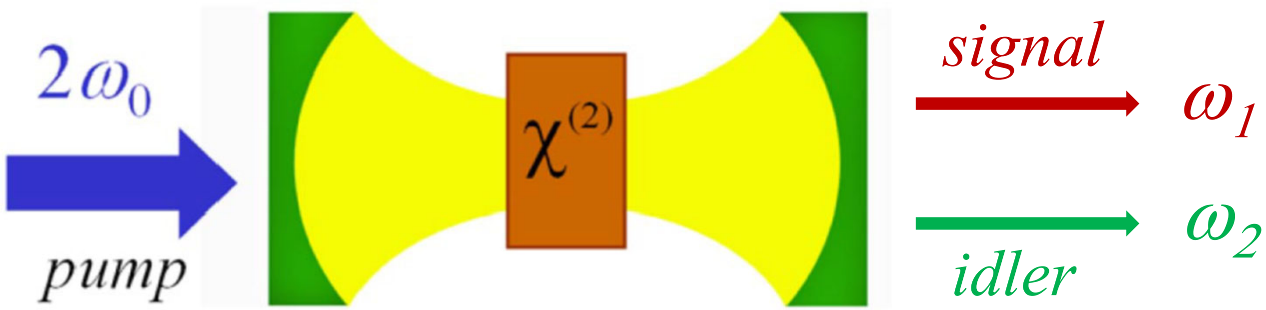
Best sources of quadrature squeezed light.

Maximum squeezing at threshold

Above threshold emmits twin beams, which are entangled.

No noise in the intensity difference.

ND-OPO



$$\omega_1 + \omega_2 = 2\omega_0$$

Hence:
- DOPO gives squeezing
- NDOPO gives entanglement
- Can be connected?

$$\hat{U} = \exp\left[i\frac{\pi}{4}\left(\hat{a}_0^\dagger \hat{a}_1 + \hat{a}_0 \hat{a}_1^\dagger\right)\right]$$

beam-splitter

$$\hat{S}_2\left(\xi\right) \longrightarrow \hat{S}_a\left(i\xi\right)\hat{S}_b\left(i\xi\right)$$

The PDC + BS Hamiltonian

PDC: parametric down-conversion
BS: beam-spliter

$$\hat{H}_e = i\hbar\left[g_1\left(\hat{a}_s^\dagger\hat{a}_i^\dagger - \hat{a}_s\hat{a}_i\right) + g_2\left(\hat{a}_s^\dagger\hat{a}_i - \hat{a}_i^\dagger\hat{a}_s\right)\right]$$

$$\hat{x}_j = \hat{a}_j^\dagger + \hat{a}_j, \quad \hat{y}_j = i\left(\hat{a}_j^\dagger - \hat{a}_j\right), \quad j = s, i,$$

$$\begin{aligned} \hat{H}_e &= \hbar\left(G\hat{x}_i\hat{y}_s - g\hat{x}_s\hat{y}_i\right), \\ G &\equiv \frac{g_1+g_2}{2}, \quad g \equiv \frac{g_2-g_1}{2}, \end{aligned}$$

Heisenberg equations

$$\frac{d}{dt}\hat{x}_i = -2g\hat{x}_s, \quad \frac{d}{dt}\hat{y}_i = -2G\hat{y}_s,$$
$$\frac{d}{dt}\hat{x}_s = 2G\hat{x}_i, \quad \frac{d}{dt}\hat{y}_s = 2g\hat{y}_i,$$

$$\Omega \equiv \sqrt{4gG} = \sqrt{g_2^2 - g_1^2}$$

$$\hat{x}_i(t) = \hat{x}_{i0}\cos\Omega t - \sqrt{\frac{g}{G}}\hat{x}_{s0}\sin\Omega t,$$
$$\hat{y}_i(t) = \hat{y}_{i0}\cos\Omega t - \sqrt{\frac{G}{g}}\hat{y}_{s0}\sin\Omega t,$$
$$\hat{x}_s(t) = \hat{x}_{s0}\cos\Omega t + \sqrt{\frac{G}{g}}\hat{x}_{i0}\sin\Omega t,$$
$$\hat{y}_s(t) = \hat{y}_{s0}\cos\Omega t + \sqrt{\frac{g}{G}}\hat{y}_{i0}\sin\Omega t,$$

We concentrate in the special case $g_2>g_1$ ($G>g$), i.e., “below threshold”

The fluctuations are fully described by the correlation matrix

$$\Gamma_{si} = \begin{pmatrix} V[\hat{x}_s] & C[\hat{x}_s,\hat{y}_s] & C[\hat{x}_s,\hat{x}_i] & C[\hat{x}_s,\hat{y}_i] \\ C[\hat{y}_s,\hat{x}_s] & V[\hat{y}_s] & C[\hat{y}_s,\hat{x}_i] & C[\hat{y}_s,\hat{y}_i] \\ C[\hat{x}_i,\hat{x}_s] & C[\hat{x}_i,\hat{y}_s] & V[\hat{x}_i] & C[\hat{x}_i,\hat{y}_i] \\ C[\hat{y}_i,\hat{x}_s] & C[\hat{y}_i,\hat{y}_s] & C[\hat{y}_i,\hat{x}_i] & V[\hat{y}_i] \end{pmatrix},$$

where

$$C\left[\hat{a},\hat{b}\right] = \frac{1}{2}\left\langle \hat{a}\hat{b} + \hat{b}\hat{a}\right\rangle - \langle\hat{a}\rangle\left\langle\hat{b}\right\rangle, \quad V[\hat{a}] = \left\langle\hat{a}^2\right\rangle - \langle\hat{a}\rangle^2 .$$

In our case

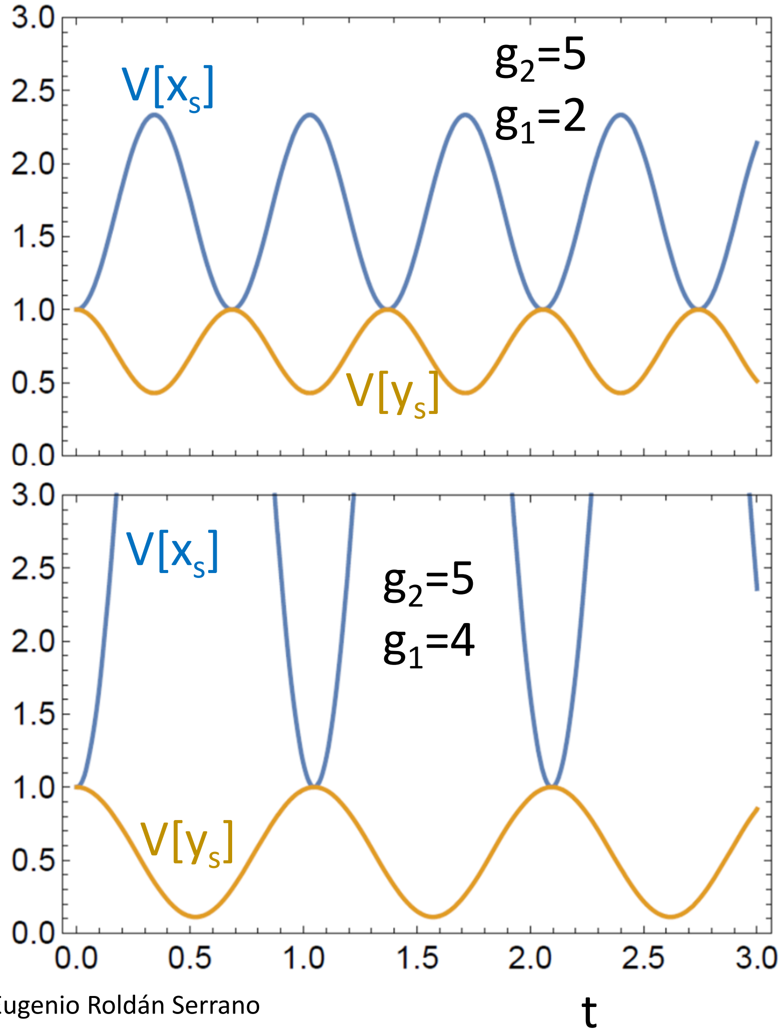
$$\Gamma_{si} = \begin{pmatrix} V[\hat{x}_s] & 0 & C[\hat{x}_s, \hat{x}_i] & 0 \\ 0 & V[\hat{y}_s] & 0 & -C[\hat{x}_s, \hat{x}_i] \\ C[\hat{x}_s, \hat{x}_i] & 0 & V[\hat{y}_s] & 0 \\ 0 & -C[\hat{x}_s, \hat{x}_i] & 0 & V[\hat{x}_s] \end{pmatrix}$$

$$V[\hat{x}_s] = 1 + \frac{2g_1}{g_2 - g_1}\sin^2\left(\sqrt{g_2^2 - g_1^2}t\right),$$

$$V[\hat{y}_s] = 1 - \frac{2g_1}{g_2 + g_1}\sin^2\left(\sqrt{g_2^2 - g_1^2}t\right),$$

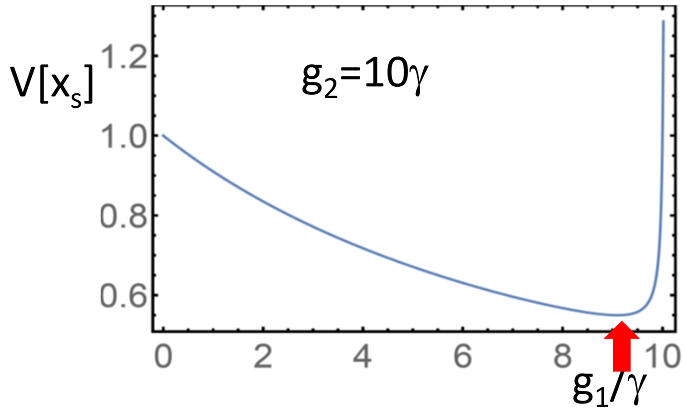
$$C[\hat{x}_s, \hat{x}_i] = \frac{g_1}{\sqrt{g_2^2 - g_1^2}}\sin\left(2\sqrt{g_2^2 - g_1^2}t\right).$$

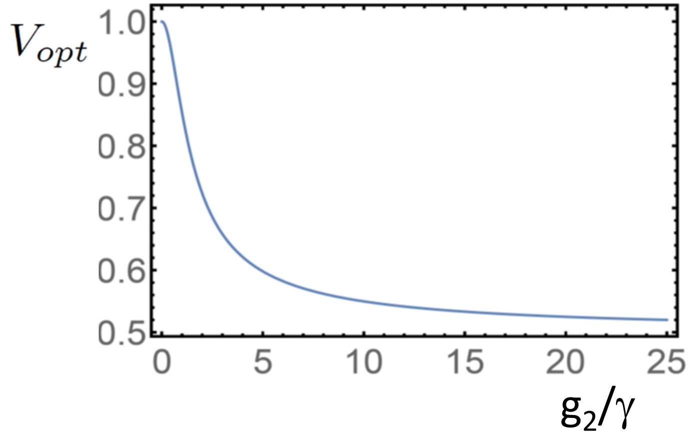
Squeezing



Influence of cavity losses

$$\langle V\left[\hat{x}_s\left(t\right)\right]\rangle \equiv 2\gamma \int_0^\infty dt\ V\left[\hat{x}_s\left(t\right)\right] e^{-2\gamma t} = \frac{\gamma^2 + g_2^2 - g_1 g_2}{\gamma^2 + g_2^2 - g_1^2}$$

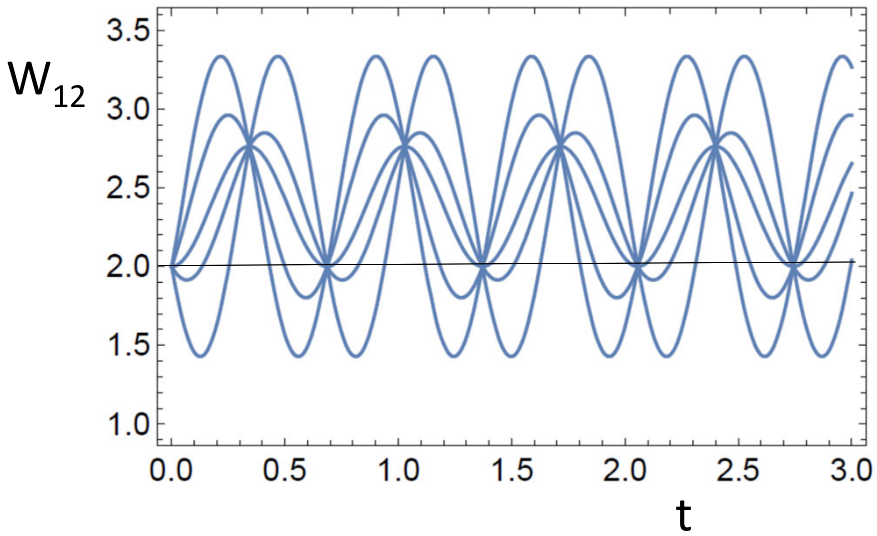




Entanglement

"Simon-Duan-Giedke-Cirac-Zoller" criterion: the states are separable iff $W_{12}(\theta)$**>**2

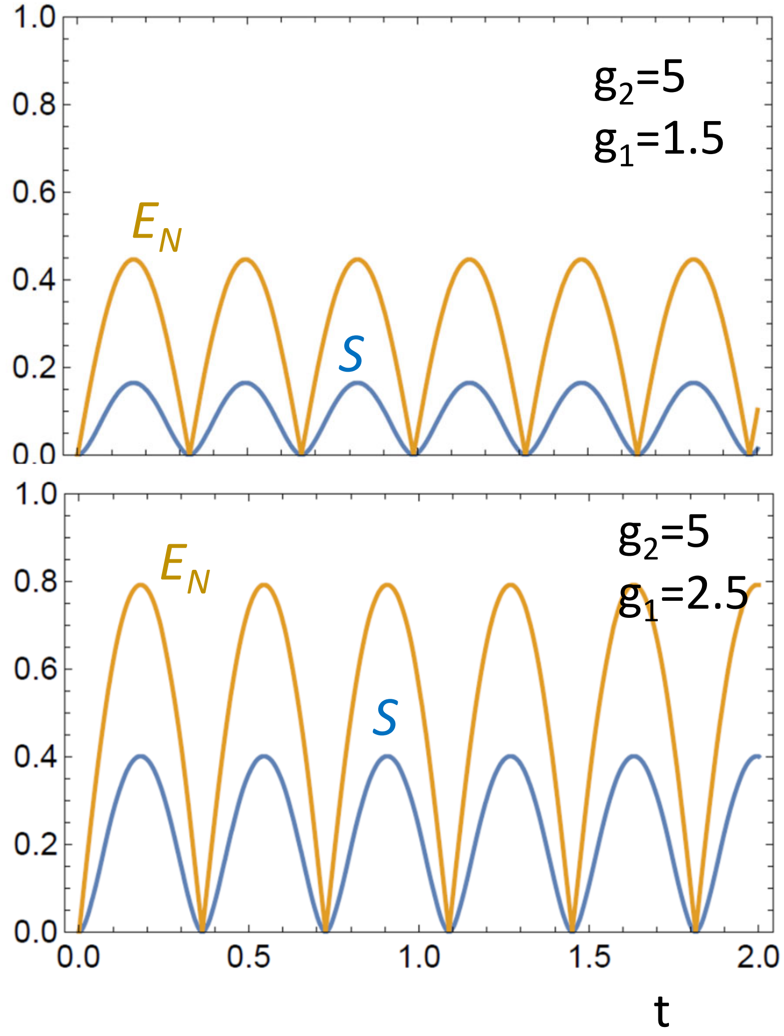
$$W_{12} = V\left[\frac{\hat{x}_1 - \hat{x}_2}{\sqrt{2}}\right] + V\left[\frac{\hat{y}_1 + \hat{y}_2}{\sqrt{2}}\right] \geqslant 2;$$



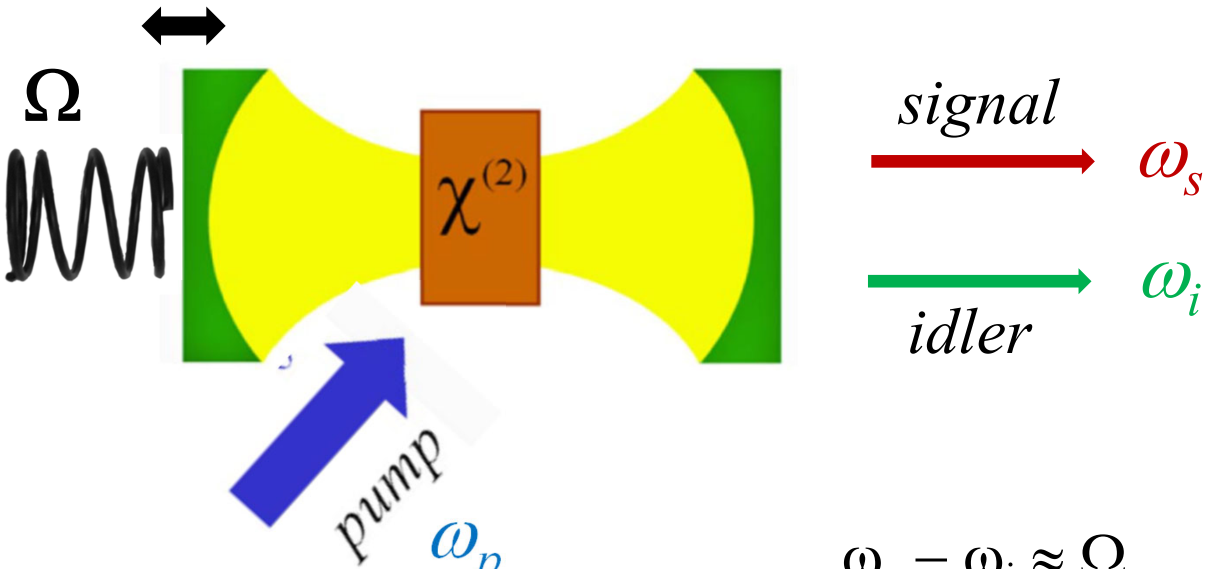
several values of θ

S: entropy

E_N: Logarithmic negativity



The non-degenerate optomechanical parametric oscillator



$$\omega_s - \omega_i \approx \Omega$$

$$\omega_s + \omega_i = \omega_p$$

The goal:
locking the signal and idler field's phases
by coupling them coherently.

Complete Hamiltonian
(Schrödinger picture)

$$
\begin{aligned}
\hat{H} &= \hat{H}_0 + \hat{H}_{PDC} + \hat{H}_{OM} + \hat{H}_{ext}, \\
\hat{H}_0 &= \hbar\omega_p \hat{a}_p^\dagger \hat{a}_p + \hbar\omega_{cs} \hat{a}_s^\dagger \hat{a}_s + \hbar\omega_{ci} \hat{a}_i^\dagger \hat{a}_i + \hbar\omega_m \hat{b}^\dagger \hat{b}, \\
\hat{H}_{PDC} &= i\hbar g_o \left(\hat{a}_p \hat{a}_s^\dagger \hat{a}_i^\dagger - \hat{a}_p^\dagger \hat{a}_s \hat{a}_i \right), \\
\hat{H}_{OM} &= -\hbar g_m \left(\hat{a}_s^\dagger \hat{a}_s + \hat{a}_i^\dagger \hat{a}_i + \hat{a}_s^\dagger \hat{a}_i + \hat{a}_i^\dagger \hat{a}_s \right) \left(\hat{b} + \hat{b}^\dagger \right), \\
\hat{H}_{ext} &= i\hbar \left(F_0 \hat{b}^\dagger e^{-i\omega_m t} - F_0^* \hat{b} e^{i\omega_m t} \right).
\end{aligned}
$$

We move to a more convenient interaction picture

$$
\begin{aligned}
\hat{a}_s &\rightarrow \hat{a}_s e^{-i\frac{\omega_p + \omega_m}{2}t}, \quad \hat{a}_i \rightarrow \hat{a}_i e^{-i\frac{\omega_p - \omega_m}{2}t}, \\
\hat{a}_p &\rightarrow \hat{a}_p e^{-i\omega_p t}, \quad \hat{b} \rightarrow \hat{b} e^{-i\omega_m t}
\end{aligned}
$$

and perform the RWA…

Model Hamiltonian
(interaction picture)

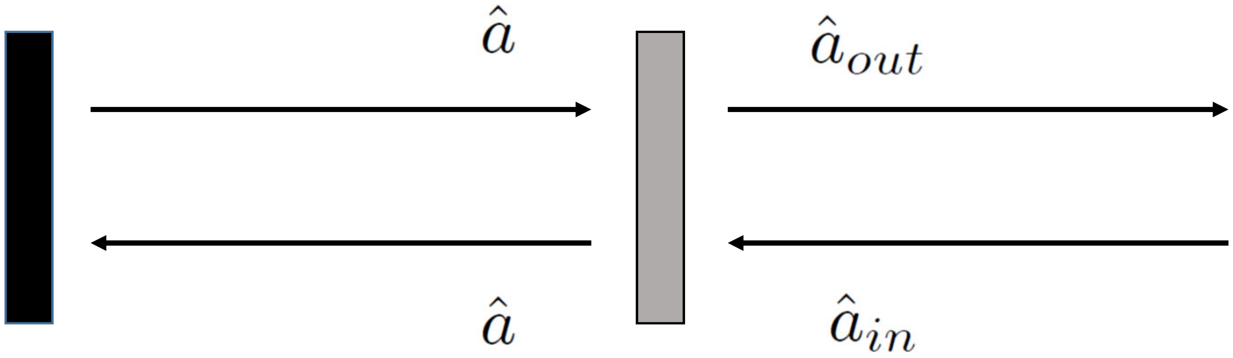
$$
\begin{aligned}
\hat{\bar{H}} &= \hat{\bar{H}}_0 + \hat{\bar{H}}_{PDC} + \hat{\bar{H}}_{OM} + \hat{\bar{H}}_{ext}, \\
\hat{\bar{H}}_0 &= -\hbar \left[\left(\delta - \frac{\Delta}{2} \right) \hat{a}_s^\dagger \hat{a}_s + \left(\delta + \frac{\Delta}{2} \right) \hat{a}_i^\dagger \hat{a}_i \right], \\
\hat{\bar{H}}_{PDC} &= i\hbar g_o \left(\hat{a}_p \hat{a}_s^\dagger \hat{a}_i^\dagger - \hat{a}_p^\dagger \hat{a}_s \hat{a}_i \right), \\
\hat{\bar{H}}_{OM} &= -\hbar g_m \left(\hat{a}_s^\dagger \hat{a}_i \hat{b} + \hat{a}_i^\dagger \hat{a}_s \hat{b}^\dagger \right) \\
\hat{\bar{H}}_{ext} &= i\hbar \left(F_0 \hat{b}^\dagger - F_0^* \hat{b} \right).
\end{aligned}
$$

$$\omega_p = \omega_{cs} + \omega_{ci} + 2\delta,$$

$$
\begin{aligned}
\Omega &\equiv \omega_{cs} - \omega_{ci}, \\
\Delta &\equiv \omega_{cs} - \omega_{ci} - \omega_m = \Omega - \omega_m,
\end{aligned}
$$

We must incorporate the coupling with the "rest of the universe" through the output mirror.

Input-output formalism.



$$a_{\mathrm{out}}(t) = -a_{\mathrm{in}}(t) + \sqrt{2\gamma}a(t)$$

$$\left[a_{\mathrm{in}}(t'), a_{\mathrm{in}}^{\dagger}(t'')\right] = \delta(t'-t'') \qquad \left[a_{\mathrm{out}}(t), a_{\mathrm{out}}^{\dagger}(t')\right] = \delta(t-t')$$

Heisenberg-Langevin equations

$$\begin{aligned}
\frac{d}{dt}\hat{a}_s &= -\left(\gamma_s - i\delta_s\right)\hat{a}_s - \alpha^2 g_o \hat{a}_s \hat{a}_i^\dagger \hat{a}_i + i g_m \hat{a}_i \hat{b} \\
&\quad + \alpha g_0 \hat{a}_{p,in} \hat{a}_i^\dagger + \sqrt{2\gamma_s}\,\hat{a}_{s,in} + \alpha g_0 E_{in} \hat{a}_i^\dagger, \\
\frac{d}{dt}\hat{a}_i &= -\left(\gamma_i - i\delta_i\right)\hat{a}_i - \alpha^2 g_o \hat{a}_i \hat{a}_s^\dagger \hat{a}_s + i g_m \hat{a}_s \hat{b}^\dagger \\
&\quad + \alpha g_0 \hat{a}_{p,in} \hat{a}_s^\dagger + \sqrt{2\gamma_i}\,\hat{a}_{i,in} + \alpha g_0 E_{in} \hat{a}_s^\dagger, \\
\frac{d}{dt}\hat{b} &= F_0 - \frac{\gamma_m}{2}\hat{b} + i g_m \hat{a}_i^\dagger \hat{a}_s + \sqrt{\frac{\gamma_m}{2}}\,\hat{b}_{in},
\end{aligned}$$

$$\delta_{s,i} = \delta \pm \frac{\Delta}{2}.$$

$$\begin{aligned}
\left\langle \hat{a}_{m,in}(t), \hat{a}_{n,in}^\dagger(t') \right\rangle &= \delta_{mn}\delta(t-t'), \quad m,n = s,i, \\
\left\langle \hat{b}_{in}(t), \hat{b}_{in}^\dagger(t') \right\rangle &= (2n_{th}+1)\,\delta(t-t').
\end{aligned}$$

$$\begin{aligned}
\left\langle \hat{a}_{p,in}^{(+)}(t) \right\rangle &= E_0, \ \left\langle \hat{a}_{p,in}^{(-)} \right\rangle = 0 \\
\left\langle \hat{a}_{p,in}^{(\pm)}(t), \hat{a}_{p,in}^{(\pm)\dagger}(t') \right\rangle &= \delta(t-t'),
\end{aligned}$$

Linearization below threshold

$$\hat{a}_j = \delta\hat{a}_j, \quad j = s, i,$$
$$\hat{b} = \bar{b} + \delta\hat{b},$$

$$\frac{d}{dt}\overrightarrow{\hat{\pi}} = \hat{\mathcal{L}} \cdot \overrightarrow{\hat{\pi}} + \sqrt{2\gamma}\vec{\eta}, \tag{24a}$$

$$\frac{d}{dt}\delta\hat{b} = -\frac{\gamma_m}{2}\delta\hat{b} + \sqrt{\frac{\gamma_m}{2}}\hat{b}_{in}, \tag{24b}$$

with $\overrightarrow{\hat{\pi}} = \mathrm{col}\left(\delta\hat{a}_s, \delta\hat{a}_s^\dagger, \delta\hat{a}_i, \delta\hat{a}_i^\dagger\right)$, $\vec{\eta} = \mathrm{col}\left(\hat{a}_{s,in}, \hat{a}_{s,in}^\dagger, \hat{a}_{i,in}, \hat{a}_{i,in}^\dagger\right)$, and the linear operator $\hat{\mathcal{L}}$ given by

$$\hat{\mathcal{L}} = \begin{pmatrix} -(\gamma - i\delta_s) & 0 & ig_m\bar{b} & \alpha g_0 E_{in} \\ 0 & -(\gamma + i\delta_s) & \alpha g_0 E_{in}^* & -ig_m\bar{b}^* \\ ig_m^*\bar{b}^* & \alpha g_0 E_{in} & -(\gamma - i\delta_i) & 0 \\ \alpha g_0 E_{in}^* & -ig_m\bar{b} & 0 & -(\gamma + i\delta_i) \end{pmatrix}. \tag{25}$$

This can be readily solved for obtaining

$$\langle\hat{\pi}_p(t)\,\hat{\pi}_q(t)\rangle \qquad \langle\hat{\pi}_p(t)\,\hat{\pi}_q(t+\tau)\rangle$$

Intracavity fluctuations (below threshold)

$$V_q^{(\theta)} = \langle \hat{a}_q(t)\, \hat{a}_q^\dagger(t) \rangle + \langle \hat{a}_q^\dagger(t)\, \hat{a}_q(t) \rangle + \langle \hat{a}_q^2(t) \rangle e^{2i\theta} + \langle \hat{a}_q^{\dagger 2}(t) \rangle e^{-2i\theta},$$

$$V_{s,i}^{(\theta)} = \frac{1 + \bar{B}^2 + \Delta^2 - \bar{B}\sigma\left(\sin 2\theta \pm \Delta \cos 2\theta\right)}{\bar{B}^2 - \left(1 + \Delta^2\right)\left(\sigma^2 - 1\right)}$$

B: mirror's forcing

σ: optical injection

Δ: detuning

$$\bar{B} \equiv \frac{g_m}{\gamma}\bar{b},$$

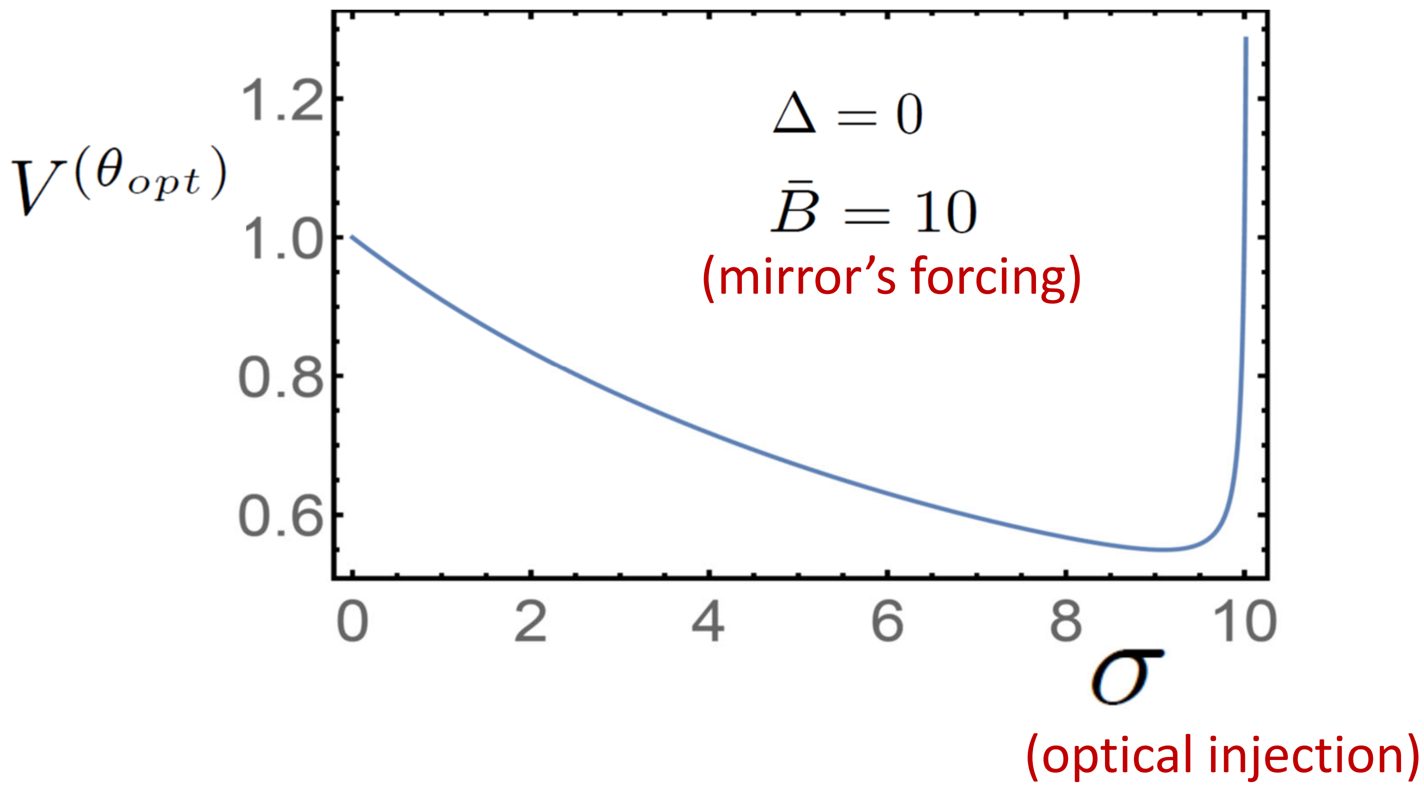
$$\sigma = \frac{\alpha g_0}{\gamma} E_{in},$$

$$\Delta_j = \frac{\delta_j}{\gamma},$$

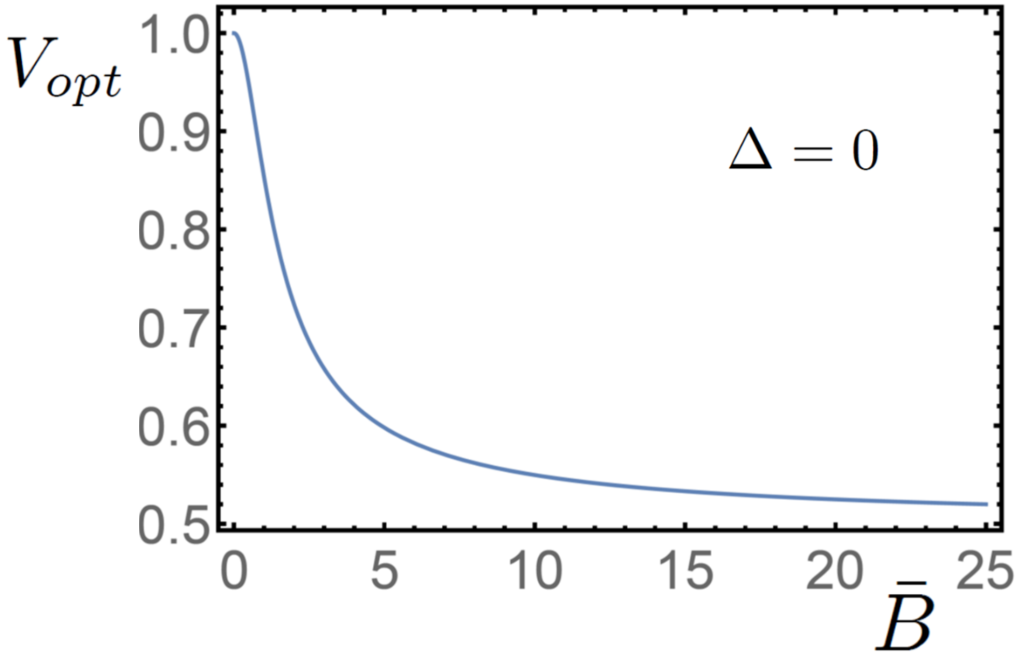
$$2\theta_q^{opt} = \begin{cases} \pi - \arctan\left(1/\Delta\right), & q = s \\ -\pi - \arctan\left(1/\Delta\right), & q = i \end{cases}$$

$$V^{(\theta_{opt})} = \frac{1 + \Delta^2 + \bar{B}^2 - \bar{B}\sigma\sqrt{1 + \Delta^2}}{\bar{B}^2 - \left(1 + \Delta^2\right)\left(\sigma^2 - 1\right)},$$

$$V^{(\theta_{opt})} = \frac{1 + \Delta^2 + \bar{B}^2 - \bar{B}\sigma\sqrt{1 + \Delta^2}}{\bar{B}^2 - (1 + \Delta^2)(\sigma^2 - 1)},$$



$$V_{opt} = \frac{1}{2}\left(1 + \sqrt{\frac{1+\Delta^2}{1+\Delta^2+\bar{B}^2}}\right)$$



Squeezing spectrum (below threshold)

As for the squeezing spectra of the fields outgoing the cavity, we limit ourselves to the resonant case, having obtained that

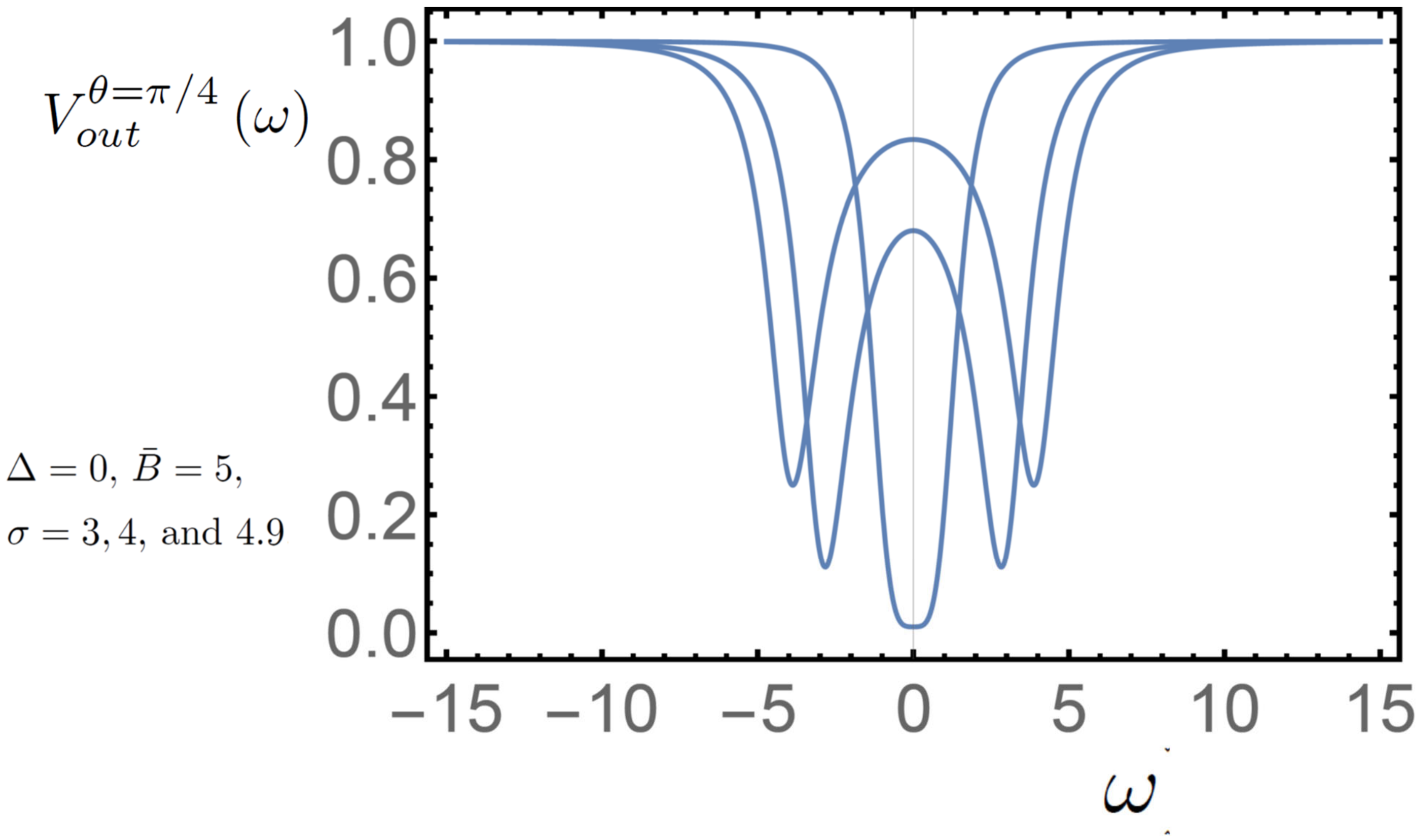
$$V_{out}(\omega) = \frac{\left(1 - \bar{B}^2 + \sigma^2 + \omega^2\right)^2 + 4\left(\bar{B}^2 + \sigma^2 - 2\bar{B}\sigma \sin 2\theta\right)}{\left(1 - \bar{B}^2 + \sigma^2 + \omega^2\right)^2 + 4\left(\bar{B}^2 - \sigma^2\right)}, \tag{52}$$

that is minimum for $\theta = \pi/4$,

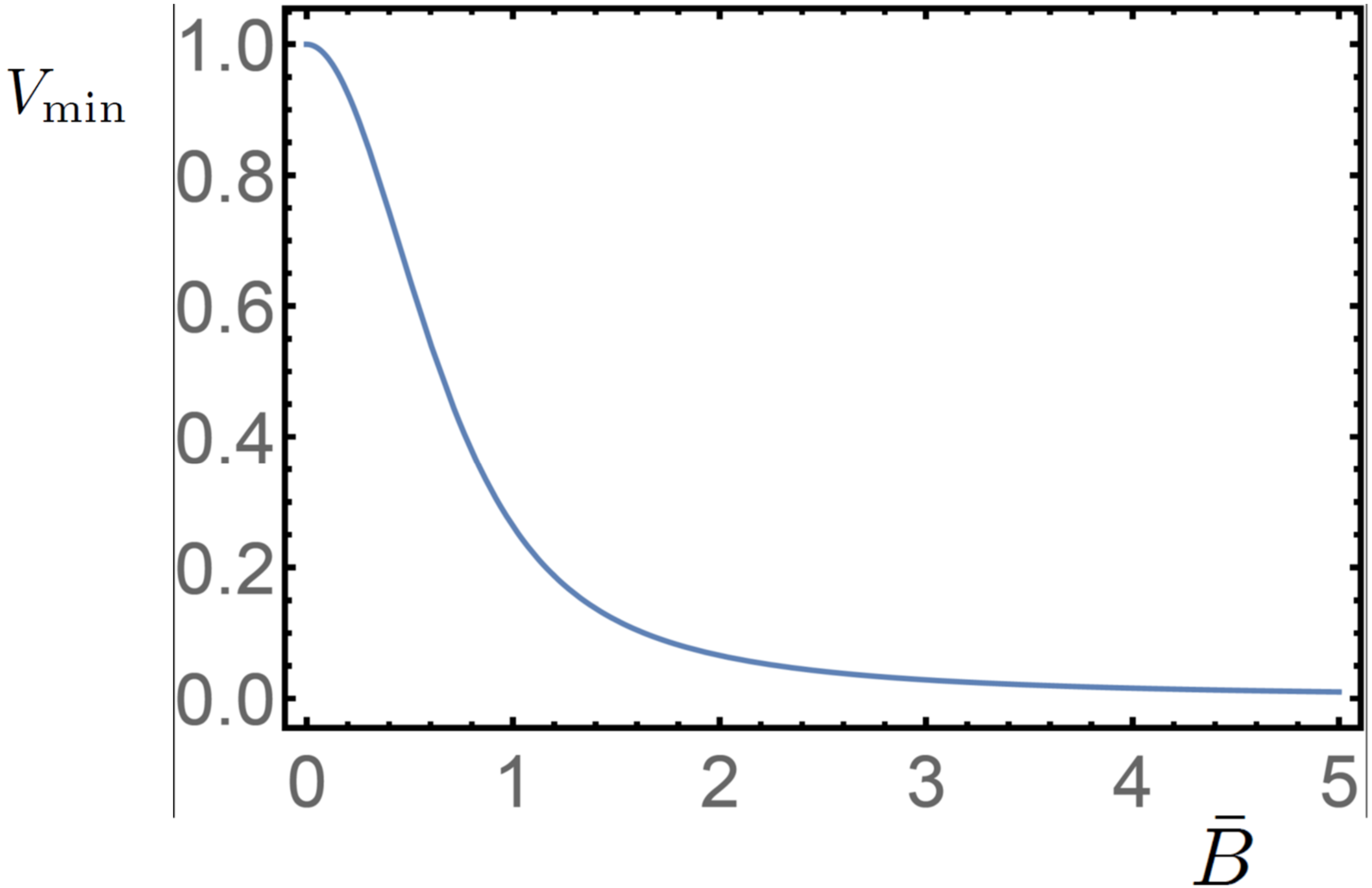
$$V_{out}^{\theta=\pi/4}(\omega) = 1 - \frac{8\sigma\left(\bar{B} - \sigma\right)}{\left(1 - \bar{B}^2 + \sigma^2 + \omega^2\right)^2 + 4\left(\bar{B}^2 - \sigma^2\right)}. \tag{53}$$

The minimum of $V_{out}^{\theta=\pi/4}(\omega)$ occurs for $\omega^2 = \bar{B}^2 - 1 - \sigma^2$ (while $\sigma \leq \sqrt{\bar{B}^2 - 1}$) and has a value

$$V_{\min} = \frac{\bar{B} - \sigma}{\bar{B} + \sigma} \underset{\sigma=\sqrt{\bar{B}^2-1}}{\Longrightarrow} \frac{\bar{B} - \sqrt{\bar{B}^2 - 1}}{\bar{B} + \sqrt{\bar{B}^2 - 1}}, \tag{54}$$


$V_{out}^{\theta=\pi/4}(\omega)$
$\Delta = 0,\ \bar{B} = 5,$
$\sigma = 3, 4,$ and 4.9
1.0
0.8
0.6
0.4
0.2
0.0
−15
−10
−5
0
5
10
15
ω

$\theta = \pi/4$ and $\Delta = 0$



Conclusions

The coupling of the idler and signal modes through the oscillations of the mirror of an optomechanical cavity effectively locks the two modes.

This results, in particular, in large levels of squeezing below threshold (the larger the squeezing the larger the mechanical forcing)

These results suggest that other means could be similarly efficient (electro-optic coupling, idler-signal coupling thorugh additional chi(2) or chi(3) processes, etc.)

Thnx!