Open Quantum Systems, Morning Coffee

Probing the strong-coupling limit of single-atom QED in an operational and contextual manner

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Advanced Physics Course on Open Quantum Systems

Selected Bibliography (textbooks)

1. **Topics in the Theory of Random Noise** Vol. 1, R. L. Stratonovich (Gordon and Breach, 1963).

2. **Photoelectron Statistics**, B. Saleh (Springer, 1978).

3. **An Open Systems Approach to Quantum Optics**, H. J. Carmichael (Springer, 1993).

4. **Statistical Methods in Quantum Optics**, Vols. 1 and 2, H. J. Carmichael (Springer, 1999 and 2008).

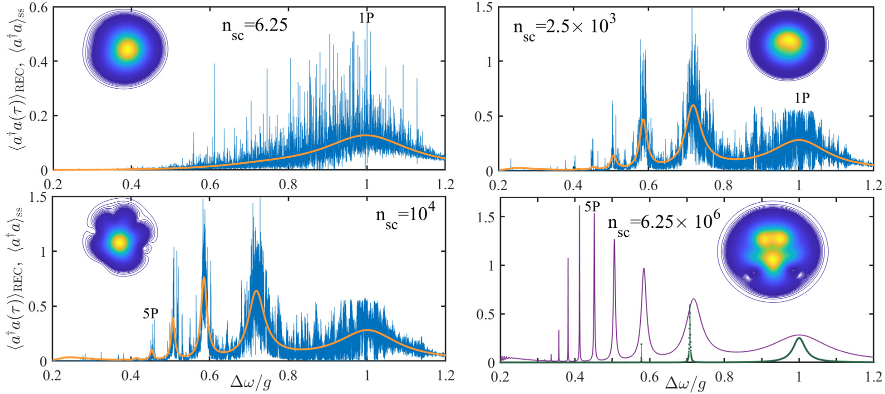
5. **The Theory of Open Quantum Systems**, H-P. Breuer and F. Petruccione (OUP, 2002).

6. **Handbook of Stochastic Methods**: for Physics, Chemistry and the Natural Sciences, C. Gardiner (Springer 2004).

7. **Quantum Statistical Properties of Radiation**, W. H. Louisell (Wiley, 1973).

A single-atom "thermodynamic limit" in QED: quantum criticality and complementarity

Persisting photon blockade for growing $n_{\rm sc} = [g/(2\kappa)]^2$ in:
(i) single trajectories, **(ii)** ensemble averages, and **(iii)** *quasi*probabilities



Motivation

- Particle statistics in a resonant environment:

  Bosons and fermions generally behave in different ways because only fermions obey the Pauli exclusion principle, which states that no two fermions may occupy the same quantum state.

  Bosons are actually attracted to the same state via the process of stimulated emission. Under some conditions, however, analogies between bosonic and fermionic behavior arise. One such condition is *photon blockade*.

- **Out-of-equilibrium transitions** and the concept of a **"thermodynamic limit"** in the absence of a conserved particle number.

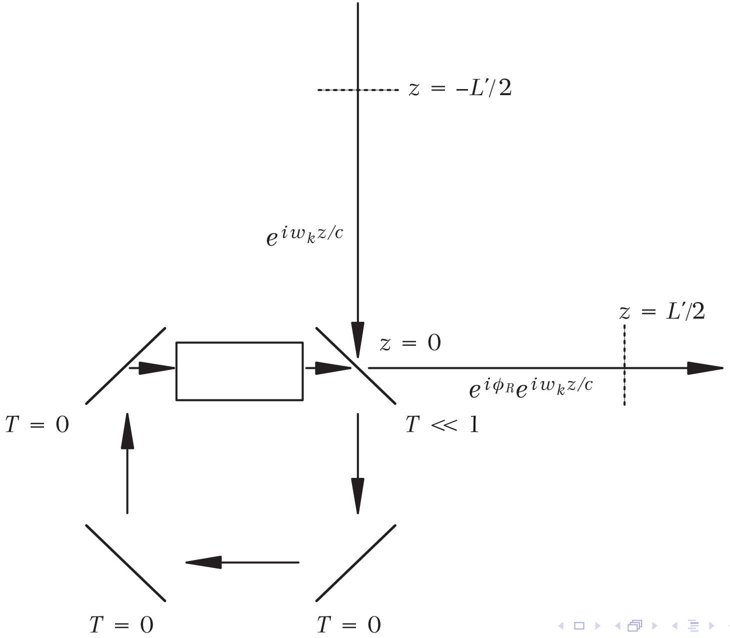
- Rethink of the physical behavior influenced in a fundamental way by inputs and outputs where internal coupling is strong enough to produce **energy-level splittings in excess of the level widths**. [1]

[1] see H. J. Carmichael, *Breakdown of Photon Blockade: A Dissipative Quantum Phase Transition in Zero Dimensions*, Phys. Rev. X **5** (2015).

Brief Outline of Concepts and Tools

- The Jaynes-Cummings (JC) model and the $\sqrt{n}$ oscillator:
  - I. Resonant excitation (absorptive bistability, spontaneous dressed-state polarization)
  - II. Off-resonance excitation (multiphoton resonances, collapse of photon blockade via complex amplitude bistability).
- Mean-field results for amplitude and phase bistability, Maxwell-Bloch equations, neoclassical equations.
- Dissipative quantum phase transitions and the two associated "thermodynamic limits":
  - I. Weak-coupling limit (laser)
  - II. Strong-coupling limit (cavity QED, circuit QED)
- The distinct role of quantum fluctuations and the JC nonlinearity: quantum vs. semiclassical picture, and quantum trajectories.

A scattering scenario in quantum optics



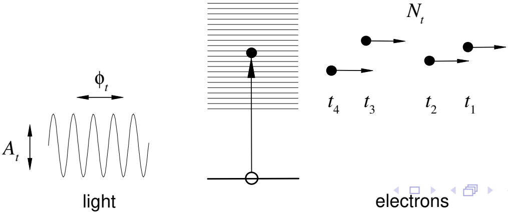
Resonance fluorescence: an open quantum system

Master equation: Source – dynamics of the atom

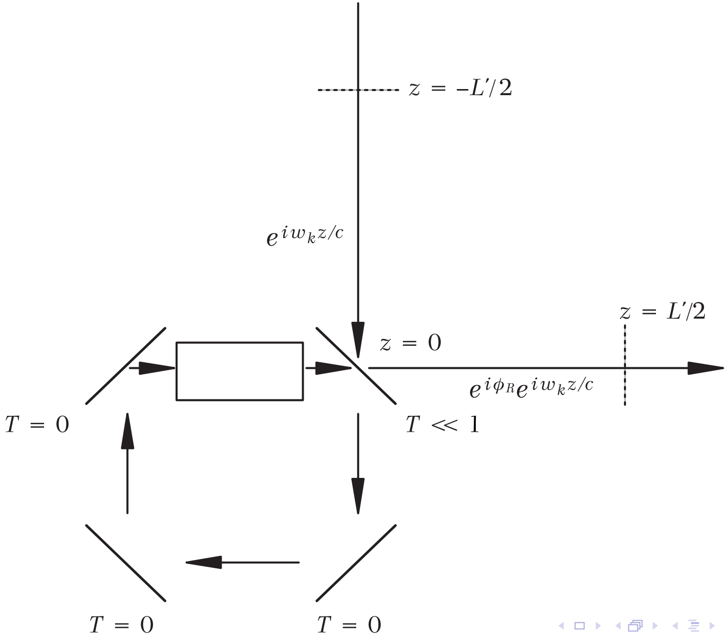
$$\dot{\rho} = -i\tfrac{1}{2}\omega_A[\sigma_z, \rho] + i(\Omega/2)[e^{-i\omega_A t}\sigma_+ + e^{i\omega_A t}\sigma_-, \rho] + \frac{\gamma}{2}(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-). \tag{1}$$

Scattered field: Detection – reservoir operators:

$$\hat{\boldsymbol{E}}_s^{(+)}(\boldsymbol{r}, t) = i\frac{1}{2\epsilon_0 V}e^{-i\omega_A t}\sum_{\boldsymbol{k},\lambda}\omega_k \hat{e}_{\boldsymbol{k},\lambda}(\hat{e}_{\boldsymbol{k},\lambda}\cdot \boldsymbol{d}_{12})e^{i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}_A)} \times \int_0^t dt' \tilde{\sigma}_-(t')e^{i(\omega_k-\omega_A)(t'-t)}. \tag{2}$$



A scattering scenario in quantum optics



Fokker–Planck equation for the laser: quantum probability within a classical stochastic description

Glauber-Sudarshan Representation: diagonal expansion in coherent states

$$\rho_L(t) = \int_{-\infty}^{\infty} d^2\alpha P_L(\alpha, \alpha^*, t) \, |\alpha\rangle\langle\alpha|, \qquad (3)$$

with P_L a positive-definite function obeying[a]

$$\kappa^{-1}\frac{\partial P_L}{\partial t} = \left[\left(\frac{\partial}{\partial\alpha}\alpha + \frac{\partial}{\partial\alpha^*}\alpha^*\right)\left(1 - \wp - \wp\frac{|\alpha|^2}{n_{\rm sat}}\right) + 2n_{\rm spon}\frac{\partial^2}{\partial\alpha\partial\alpha^*}\right] P_L. \qquad (4)$$

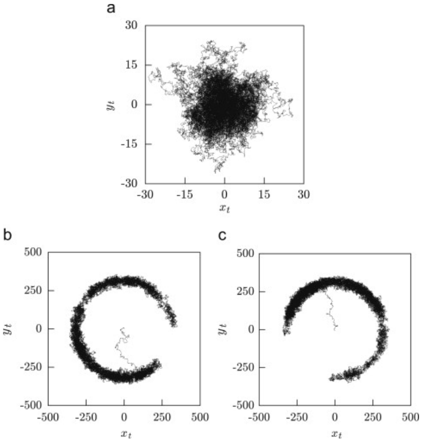
[a] F. Haake and M. Lewenstein, Phys. Rev. A **27** (1983); H.J.Carmichael and Changsuk Noh, Physica E **42** (2010)

Equivalent Ito stochastic differential equation

$$d\alpha_t = -\kappa\alpha_t\left(1 - \wp - \wp\frac{|\alpha_t|^2}{n_{\rm sat}}\right) dt + \sqrt{2\kappa n_{\rm spon}}\, dZ, \qquad (5)$$

where dZ is a complex Wiener increment satisfying
$\overline{dZ\,dZ} = \overline{dZ^*\,dZ^*} = 0$ and $\overline{dZ^*\,dZ} = dt$.

Realizations of a classical stochastic process



Sample paths in the complex plane, $\alpha_t = x_t + iy_t$, from numerical simulation of Eq. (3): (a) below threshold, $1 - \wp = 10^{-2}$; (b) and (c) above threshold, $\wp - 1 = 10^{-3}$; for $n_{\rm sat} = 10^8$ and $n_{\rm spon} = 1$.

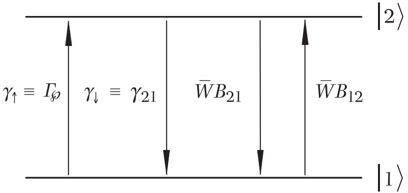
Mean amplitude scales as $\overline{|\alpha_t|} = \sqrt{n_{\rm sat}(\wp - 1)}$. The paths plotted in frames (b) and (c) of play out over 10^6 cavity lifetimes, yet the phase has still not diffused a full 2π.

Transition from $g^{(2)}(\tau) = 1 + \exp[-2\kappa(1-\wp)|\tau|]$ to $g^{(2)}(\tau) = 1 + \frac{n_{\rm spon}}{n_{\rm sat}(\wp-1)^2}\exp[-2\kappa(1-\wp)|\tau|]$ across threshold[2].

[2] H.J.Carmichael and Changsuk Noh, Physica E **42** (2010).

Laser operation: some particulars of a weak-coupling limit

Effective two-level model[3] for the laser medium ($g = \sqrt{\frac{\omega_C d^2}{2\hbar\epsilon_0 V_Q}}$):



$$n_{\rm sat} = n_{wc} = \frac{\gamma_h(\gamma_\uparrow + \gamma_\downarrow)}{8g^2},$$

$$\wp = \frac{2g^2}{\gamma_h\kappa}(N_2^0 - N_1^0) = 2C\frac{\gamma_\uparrow - \gamma_\downarrow}{\gamma_\uparrow + \gamma_\downarrow}.$$

The strength of the fluctuations is determined by two intensive parameters: the thermal photon number

$$\bar{n} = \left[e^{\hbar\omega_C/(k_B T)} - 1\right]^{-1}, \qquad (7)$$

and the spontaneous emission photon number

$$n_{\rm spon} = C + \tfrac{1}{2}\wp = \frac{2Ng^2}{\gamma_h\kappa}\frac{\gamma_\uparrow}{\gamma_\uparrow + \gamma_\downarrow}. \quad \text{For } \gamma_\uparrow \gg \gamma_\downarrow,\, n_{\rm spon} \approx \wp. \qquad (8)$$

Weak-coupling limit: $g^2 \to 0$, $N_2^0 \to \infty$, while $C \approx N_2^0 g^2/(\gamma_h\kappa) = \text{const}$.

[3] H. J. Carmichael, Statistical Methods in Quantum Optics, 1, Springer 1999

Master Equation, Wigner and Q representations

- The Master Equation describing the evolution of ρ for a cavity mode coupled to a two-level atom with strength g in the presence of dissipation (at rates 2κ for the cavity photons and γ, γ_ϕ for the atom) is:

$$\boxed{\begin{aligned}\dot{\rho} = &-(i/\hbar)[H_{JC}, \rho] + \kappa[\overline{n}(\omega_c)+1]\mathcal{L}\{a, \rho\} + \kappa\overline{n}(\omega_c)\mathcal{L}\{a^\dagger, \rho\} + \\ &+ (\gamma/2)[\overline{n}(\omega_A)+1]\mathcal{L}\{\sigma_-, \rho\} + (\gamma/2)\overline{n}(\omega_A)\mathcal{L}\{\sigma_+, \rho\} + \\ &+ (\gamma_\phi/2)\mathcal{L}\{\sigma_z, \rho\},\end{aligned}} \tag{9}$$

where $\mathcal{L}\{B, \rho\} = 2B\rho B^\dagger - B^\dagger B\rho - \rho B^\dagger B$ and $\overline{n}(\omega_c)$ is the photon number of a bath oscillator in thermal equilibrium (temperature T) and frequency ω_C. **Strong-coupling regime:** $g \gg 2\kappa, \gamma$.

- Characteristic functions:

$$\chi_A(z, z^*) = \mathrm{tr}(\rho e^{iza} e^{iz^* a^\dagger}) \quad \text{and} \quad \chi_S(z, z^*) = \mathrm{tr}(\rho e^{iza + iz^* a^\dagger}). \tag{10}$$

- Their Fourier transforms

$$Q, W(\alpha, \alpha^*) = \int \chi_{A,S}(z, z^*) e^{-iz^*\alpha^*} e^{-iz\alpha}\, d^2 z, \tag{11}$$

are quasi-probability density functions.

The driven Jaynes-Cummings model

$$\boxed{H_{JC} = \frac{1}{2}\hbar\omega_A\sigma_z + \hbar\omega_c a^\dagger a + i\hbar g\left(a^\dagger\sigma_- - a\sigma_+\right) + \hbar\left(\varepsilon_d e^{-i\omega_d t}a^\dagger + \varepsilon_d^* e^{i\omega_d t}a\right).} \tag{12}$$

- Two competing interactions: the JC interaction between the atom and the cavity mode, and the interaction of the cavity mode with the external driving field. [4]
- In **resonance fluorescence** the bare atomic levels split as a result of the atom-field interaction:

$$H_{\text{res.fl.}} = \frac{1}{2}\hbar\omega_A\sigma_z + \hbar\omega_c a^\dagger a + \hbar\left(\lambda a\sigma_+ + \lambda^* a^\dagger\sigma_-\right) \tag{13}$$

- The new energies of the dressed states are

$$E_{n,\pm} = \left(n + \frac{1}{2}\right)\hbar\omega_A \pm \sqrt{n+1}\,\hbar\left|\lambda\right|. \tag{14}$$

- With driving we expect 'dressing' of the 'dressed states'. The threshold for spontaneous polarization occurs at $2\left|\varepsilon_d\right| = g$.

[4] H. J. Carmichael, Statistical Methods in Quantum Optics, 2, Springer 2008

Resonance – Quasi-energy spectrum I.

- Quasi-energies are associated with a time-independent Schrödinger equation with Hamiltonian:

$$\tilde{H} = i\hbar g \left(a^{\dagger}\sigma_{-} - a\sigma_{+}\right) + \hbar \left(\varepsilon_d a^{\dagger} + \varepsilon_d^{*} a\right) \tag{15}$$

- For strong driving fields ($2\left|\varepsilon_d\right|/g \gg 1$) the roles of the interactions are reversed: the JC interaction becomes the perturbation.

- Potential energy $\hbar\varepsilon_d(a^{\dagger} + a)$ proportional to the position of a harmonic oscillator. **We anticipate a continuous spectrum** with a transition located at $2\left|\varepsilon_d\right| = g$ where the quasi-energy spectrum collapses. That point is identified as the organizer of a second-order dissipative quantum phase transition in the presence of quantum fluctuations.
- With drive and cavity detuning, the large mean photon number is

$$n_{\rm ss} = \frac{\left|\varepsilon_d\right|^2}{\kappa^2 + \left[(\omega_d - \omega_c) \mp g/(2\sqrt{n_{\rm ss}})\right]^2}. \tag{16}$$

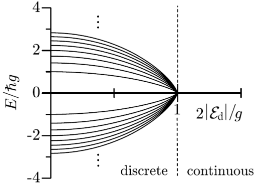
Resonance – Quasi-energy spectrum II.

- Define a squeeze parameter (think of the parametric oscillator) [a]

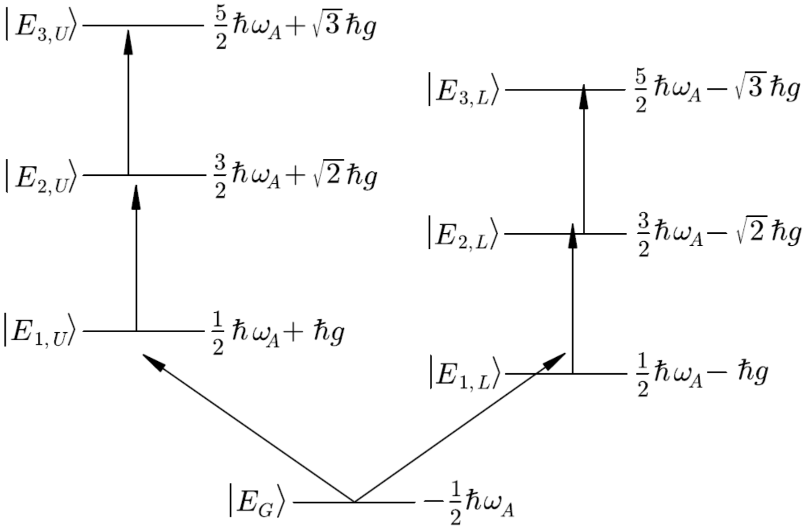
$$e^{-2r} \equiv \sqrt{1-(2\left|\varepsilon_d\right|/g)^2}. \qquad (17)$$

- Quasi-energies $E_{n,U(L)} = \pm\sqrt{n}\,\hbar g e^{-3r}$.
- Spectrum $(m-\frac{1}{2})\omega_A + E_{n,\pm}/\hbar$, $m = 0, 1, 2, \ldots$ below threshold.
- At threshold, the discrete quasi-energy levels merge into a continuum. Steady states are formed from superpositions of atomic states multiplied by squeezed and displaced Fock states. Above threshold, normalizable states do not exist.

[a] P. Alsing *et al.*, *Dynamic Stark effect for the Jaynes-Cummings system*, Phys. Rev. A **45** (1992).



Resonance —The two ladders of eigenstates I.



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[5] H. J. Carmichael, Statistical Methods in Quantum Optics 2, 2008

Resonance – The two ladders of eigenstates II.

- Two quasi-annihilation operators U and L for two ladders beginning from the same ground state.

- The JC Hamiltonian can be written as $(\omega_A = \omega_c = \omega_d)$:

$$\begin{aligned} H_S + \tfrac{1}{2}\hbar\omega_A = 0\,|G\rangle\,\langle G| + \left(\hbar\omega_A U^\dagger U + \hbar g\sqrt{U^\dagger U}\right) + \\ + \left(\hbar\omega_A L^\dagger L - \hbar g\sqrt{L^\dagger L}\right) + \hbar\left(\varepsilon_d a^\dagger + \varepsilon_d^* a\right) = H^+_{\sqrt{n}} + H^-_{\sqrt{n}}. \end{aligned} \tag{18}$$

- Two $\sqrt{n}$ anharmonic oscillators driven away from resonance.

- For a small cavity damping we form the Master Equation (U)-oscillator

$$\dot{\rho} = \frac{1}{i\hbar}\left[H^+_{\sqrt{n}},\, \rho\right] + \kappa\left(2U^\dagger \rho U - U^\dagger U\rho - \rho U^\dagger U\right) \tag{19}$$

Resonance – Two limits of the $\sqrt{n}$ oscillator

- Weak excitation limit ($\sqrt{n}$ oscillator):

$$\langle a^\dagger a \rangle_{ss} \approx \left| \frac{\varepsilon_d}{\kappa + ig} \right|^2 \approx \left(\frac{|\varepsilon_d|}{g} \right)^2 . \tag{20}$$

- Strong excitation, quasi-resonant, with detuning $g/(2\sqrt{n})$ (for the $\sqrt{n}$ oscillator):

$$\langle a^\dagger a \rangle_{ss} \approx \left| \frac{\varepsilon_d}{\kappa + ig/(2\sqrt{\langle a^\dagger a \rangle_{ss}})} \right|^2 \approx \left(\frac{|\varepsilon_d|}{\kappa} \right)^2 - \left(\frac{g}{2\kappa} \right)^2 . \tag{21}$$

- Mean-field equations predict above threshold – full JC oscillator:

$$|\alpha_{\rm ss}|^2 = \left(\frac{|\varepsilon_d|}{\kappa} \right)^2 \left[1 - \left(\frac{g}{2\,|\varepsilon_d|} \right)^2 \right] . \tag{22}$$

Photon counting – A damped coherent state I.

Constituents of the trajectory

The damped resonator is described by the jump operator and non-Hermitian Hamiltonian, $J = \sqrt{2\kappa}\, a$, $H = -i\hbar\kappa a^\dagger a$. We adopt the *ansatz*

$$|\overline{\psi}_{\mathrm{REC}}(t)\rangle = A(t)\,|\alpha(t)\rangle\,, \tag{23}$$

with $|\alpha(t)\rangle = \exp[\alpha(t)a^\dagger - \alpha^*(t)a]\,|0\rangle$. The norm $\langle\overline{\psi}_{\mathrm{REC}}(t)|\overline{\psi}_{\mathrm{REC}}(t)\rangle = |A(t)|^2$ is the record probability density.

Record of n counts up to time t: jumps at the ordered times $t_1, t_2, \ldots, t_n$

A jump at t_k preserves the ansatz: $A(t_k) \to \sqrt{2\kappa}\,\alpha(t_k)A(t_k)$.
The evolution between jumps obeys:

$$\frac{d\,|\overline{\psi}_{\mathrm{REC}}(t)\rangle}{dt} = -(\kappa a^\dagger a)\,|\overline{\psi}_{\mathrm{REC}}(t)\rangle\,. \tag{24}$$

This also preserves the ansatz provided that

$$\frac{d\alpha(t)}{dt} = -\kappa\alpha(t) \quad \text{and} \quad \frac{1}{A(t)}\frac{dA(t)}{dt} = \frac{d\alpha^*(t)}{dt}\alpha(t) = \frac{d}{dt}\left(\tfrac{1}{2}|\alpha(t)|^2\right),$$

Photon counting – A damped coherent state II.

Expression for the conditional state

For $t_k \le t \le t_{k+1}$: $\alpha(t) = \alpha\exp(-\kappa t)$ and $A(t) = A(t_k)\exp[-\frac{1}{2}|\alpha|^2(e^{-2\kappa t_k} - e^{-2\kappa t})]$. We can then construct:

$$|\overline{\psi}_{\rm REC}(t)\rangle = (\sqrt{2\kappa}|\alpha|^2 e^{-2\kappa t_n})\ldots(\sqrt{2\kappa}|\alpha|^2 e^{-2\kappa t_1}) \times \exp[-\tfrac{1}{2}|\alpha|^2(1 - e^{-2\kappa t})]\,|\alpha e^{-\kappa t}\rangle\,, \tag{25}$$

and find the probability density

$$\langle\overline{\psi}_{\rm REC}(t)|\overline{\psi}_{\rm REC}(t)\rangle = (2\kappa|\alpha|^2 e^{-2\kappa t_n})\ldots(2\kappa|\alpha|^2 e^{-2\kappa t_1}) \times \exp[-|\alpha|^2(1 - e^{-2\kappa t})]. \tag{26}$$

Probability for n counts in time T

By summing (integrating) over all possible times we find[a]:

$$P(n,T) = \frac{[|\alpha|^2(1 - e^{-2\kappa T})]^n}{n!}\exp[-|\alpha|^2(1 - e^{-2\kappa T})]. \tag{27}$$

[a] H.J.Carmichael, Quantum Open Systems, Ch. 4 in *Strong Light-Matter Coupling: From Atoms to Solid-State Systems*, World Scientific Publishing (2013).

Resonance – Forming a quantum trajectory from jumps I.

- The state of the cavity field obeys the stochastic equation:

$$\frac{d\tilde{\alpha}}{dt} = -[\kappa + i\epsilon g/(2|\tilde{\alpha}|)]\tilde{\alpha} + i\varepsilon_d. \qquad (28)$$

- $\epsilon = \pm 1$ represents the random phase switching instigated by a single quantum event.
- At strong excitation the JC interaction term gives rise to the operator $d_z \approx |u\rangle\langle u| - |l\rangle\langle l|$, with $|u, l\rangle = [\frac{\sqrt{2}}{2}(|+\rangle \pm i\,|-\rangle)]$. This in turn features in the coupling term[6]

$$\dot{\rho}_{q\alpha} = -ig/(2\sqrt{\overline{n}})\tfrac{1}{2}\left(d_z[a^\dagger a, \rho] + [a^\dagger a, \rho]d_z\right). \qquad (29)$$

- Performing the secular transformation yields the "switching terms"

$$\ldots + \gamma/4\,(d_-\tilde{\rho}d_+ + d_+\tilde{\rho}d_-) + \ldots \qquad (30)$$

with $d_+ = |u\rangle\langle l|$ and $d_- = |l\rangle\langle u|$

- These terms couple the U and L paths according to the system size.

[6] P. Alsing and H. J. Carmichael, Quantum Opt. **3** (1991).

Resonance - Forming a quantum trajectory from jumps II.

- Emission times: $t_1, t_2, \ldots t_N$ between which there is coherent evolution with a non-Hermitian Hamiltonian.
- $\mathcal{S}$: **collapse operator** and $e^{(L-\mathcal{S})(t_j - t_{j-1})}$: the **propagator**

$$\tilde{\rho}_c(t) = \begin{cases} \dfrac{e^{(\mathcal{L}-\mathcal{S})(t-t_{j-1})}\tilde{\rho}_c(t_{j-1})}{\mathrm{tr}[e^{(\mathcal{L}-\mathcal{S})(t-t_{j-1})}\tilde{\rho}_c(t_{j-1})]}, & t_{j-1} \leq t < t_j \\ \dfrac{\mathcal{S}e^{(\mathcal{L}-\mathcal{S})(t-t_{j-1})}\tilde{\rho}_c(t_{j-1})}{\mathrm{tr}[\mathcal{S}e^{(\mathcal{L}-\mathcal{S})(t-t_{j-1})}\tilde{\rho}_c(t_{j-1})]}, & t = t_j. \end{cases} \tag{31}$$

- with $\mathcal{S}\,O = (\gamma/4)\,(d_- O d_+ + d_+ O d_-)$ and
- $(\mathcal{L}-\mathcal{S})O$ determines evolution between switching events: independent driven harmonic oscillators along either path.

Dyson expansion for the density operator:

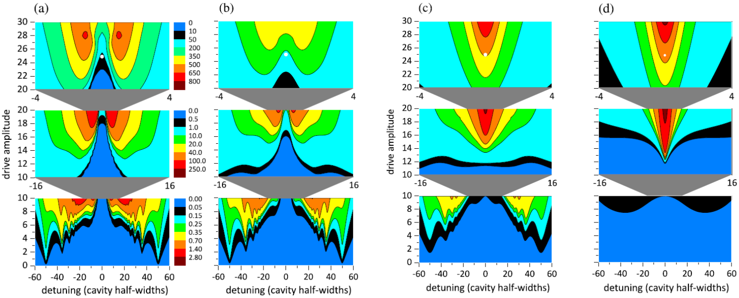
$$\overline{\rho}(t) = \sum_{k=0}^{\infty} \int_{t_0}^{t} dt_k \int_{t_0}^{t_k} dt_{k-1} \cdots \int_{t_0}^{t_2} dt_1 \; \exp[(\mathcal{L}-\mathcal{S})(t-t_k))] \cdots$$
$$\mathcal{S}\exp[(\mathcal{L}-\mathcal{S})(t_k - t_{k-1}))]\mathcal{S}\exp[(\mathcal{L}-\mathcal{S})(t_2 - t_1))] \cdots$$
$$\mathcal{S}\exp[(\mathcal{L}-\mathcal{S})(t_1 - t_0))]\overline{\rho}(t_0). \tag{32}$$

Resonance – Decoherence and ladder switching

$$a\left|E_{n,(U,L)}\right\rangle = \frac{\sqrt{n}+\sqrt{n+1}}{2}\left|E_{n-1,(U,L)}\right\rangle + \frac{\sqrt{n}-\sqrt{n+1}}{2}\left|E_{n-1,(L,U)}\right\rangle \tag{33}$$

$$\sigma_-\left|E_{n,(U,L)}\right\rangle = \tfrac{1}{2}\left|E_{n,U}\right\rangle + \tfrac{1}{2}\left|E_{n,L}\right\rangle . \tag{34}$$

(i.e. 50% probability of ladder switch). For $\gamma/\kappa = 0.1, 1, 10, 100$ [7].



[7] H. J. Carmichael, Phys. Rev. X **5**, 2015 (Fig. 7)

Resonance – Spontaneous dressed-state polarization

For slowly-varying operators with $\langle \tilde{a} \rangle = e^{i\omega_C t} \langle a \rangle$, we define [8]:
Cavity field:

$$x + iy \equiv N^{-1/2}[ie^{-i\arg(\varepsilon_d)} \langle \tilde{a} \rangle]. \tag{35}$$

Collective Bloch vector:

$$\mathbf{m} \equiv N^{-1} \left\{ 2\mathrm{Re}[ie^{-i\arg(\varepsilon_d)} \langle \tilde{J}_- \rangle]\hat{x} + 2\mathrm{Im}[ie^{-i\arg(\varepsilon_d)} \langle \tilde{J}_- \rangle]\hat{y} + \langle J_z \rangle \hat{z} \right\} \tag{36}$$

Neoclassical equations $[\gamma/(2\kappa) = 0]$:

$$\begin{aligned} \dot{x} &= -\kappa x + \tfrac{1}{2}\sqrt{N}\, g m_x + |\varepsilon_d|/\sqrt{N}, \\ \dot{y} &= -\kappa y + \tfrac{1}{2}\sqrt{N}\, g m_y, \end{aligned} \tag{37}$$

giving:

$$\dot{\mathbf{m}} = \mathbf{B} \times \mathbf{m}, \quad \mathbf{m} \cdot \mathbf{m} = 1, \tag{38}$$

with $\mathbf{B} \equiv 2\sqrt{N}\, g(-y, x, 0)$: *dynamically changing magnetic field.*
Above threshold, $\mathbf{m}_{\mathrm{ss}}$ and $\mathbf{B}_{\mathrm{ss}}$ adjust their values so that the pair either align or anti-align with each other.

[8] P. Alsing and H. J. Carmichael, Quantum Opt. **3** (1991).

Scaling in the strong-coupling “thermodynamic limit”

- Solving in the presence of detuning $\Delta\omega = \omega_d - \omega_C$ yields

$$\alpha_{\rm ss} = -i\varepsilon_d \left[\kappa - i\left(\Delta\omega \mp {\rm sgn}(\Delta\omega)\frac{g^2}{\sqrt{(\Delta\omega)^2 + 4g^2|\alpha_{\rm ss}|^2}}\right)\right]^{-1}. \tag{39}$$

- Steady state equation with scale parameter $n_{\rm sc} = g^2/(4\kappa^2)$.

$$\frac{|\alpha_{\rm ss}|^2}{n_{\rm sc}}\left[\frac{|\alpha_{\rm ss}|^2}{n_{\rm sc}} + 1 - \left(\frac{2|\varepsilon_d|}{g}\right)^2\right] = 0. \tag{40}$$

Otherwise similar to the laser state equation

$$n_{\rm ss}\left(\frac{n_{\rm ss}}{n_{\rm sat}} + 1 - \wp\right) = 0 \tag{41}$$

- If the drive is tuned to the n-photon resonance:

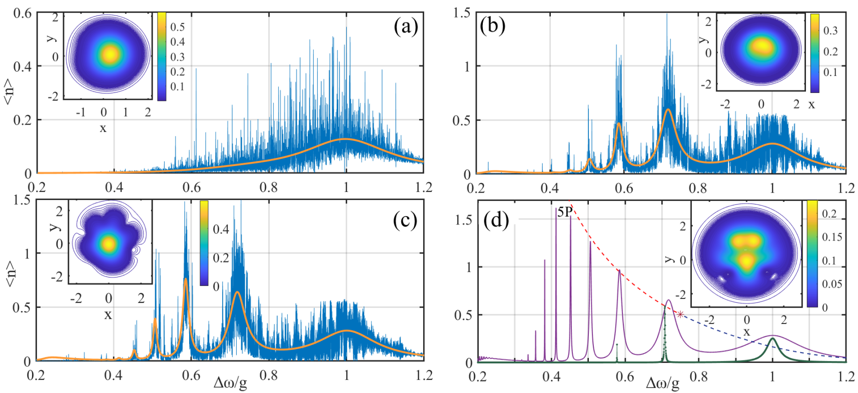
$$n\hbar\omega_d = n\hbar\omega_c \mp \sqrt{n}\,\hbar g, \quad \text{then} \tag{42}$$

$$E_{n+1,(U,L)} - E_{n,(U,L)} - \hbar\omega_d \approx \mp\sqrt{\frac{n_{\rm sc}}{n}}\,\hbar\kappa. \tag{43}$$

- The regime of photon blockade does not collapse as $n_{\rm sc} \to \infty$.

Photon blockade in the strong-coupling limit

Persistence of photon blockade with growing n_{sc} (for $|\Delta\omega| \sim g$) [9]

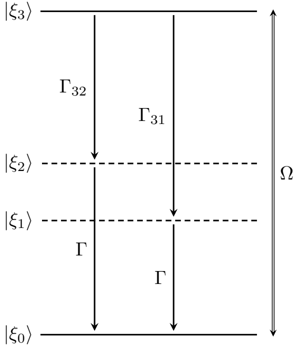


Quantum fluctuations produce a pronounced disagreement with the mean-field predictions as $n_{\mathrm{sc}} \to \infty$.

[9] Th. K. M, Phys. Rev. A **100** (2019).

Monitoring coherence in photon blockade I.

Two-photon resonance ($|\Delta\omega| \approx g/\sqrt{2}$) with $\varepsilon_d/g \ll 1$ and $\gamma = 2\kappa \ll g$:



$$|\xi_0\rangle \equiv |0,-\rangle\,,$$
$$|\xi_1\rangle \equiv \frac{1}{\sqrt{2}}(|1,-\rangle - |0,+\rangle),$$
$$|\xi_2\rangle \equiv \frac{1}{\sqrt{2}}(|1,-\rangle + |0,+\rangle),$$
$$|\xi_3\rangle \equiv \frac{1}{\sqrt{2}}(|2,-\rangle - |1,+\rangle).$$

$$\Gamma_{32} + \Gamma_{31} = 2\Gamma = 2\gamma,\ \Omega \approx 2\sqrt{2}\frac{\varepsilon_d^2}{g}.$$

$$\frac{d\rho}{dt} = \mathcal{L}\rho \equiv -i[\tilde{H}_{\text{eff}}, \rho] + \Gamma_{32}\mathcal{D}[|\xi_2\rangle\langle\xi_3|](\rho) + \Gamma_{31}\mathcal{D}[|\xi_1\rangle\langle\xi_3|](\rho) + \Gamma\mathcal{D}[|\xi_0\rangle\langle\xi_1|](\rho) + \Gamma\mathcal{D}[|\xi_0\rangle\langle\xi_2|](\rho), \tag{45}$$

$$\boxed{\tilde{H}_{\text{eff}} \equiv \sum_{k=0}^{3} \tilde{E}_k |\xi_k\rangle\langle\xi_k| + \hbar\Omega(e^{2i\omega_d t}|\xi_0\rangle\langle\xi_3| + e^{-2i\omega_d t}|\xi_3\rangle\langle\xi_0|).} \tag{46}$$

A quantum beat (freq. $\approx 2g$) is superimposed on top of a semiclassical Rabi oscillation (freq. $\approx 2\Omega$); ringing as the transition saturates.
A spontaneous-emission event after steady state prepares the mixed state:

$$\rho_{\rm cond}(0) = \frac{2}{3}|0,-\rangle\langle 0,-| + \frac{1}{3}|\psi_{\rm super}\rangle\langle\psi_{\rm super}|, \tag{47}$$

with

$$|\psi_{\rm super}\rangle = \frac{1}{\sqrt{2}}(|\xi_1\rangle + |\xi_2\rangle) = |1,-\rangle\,. \tag{48}$$

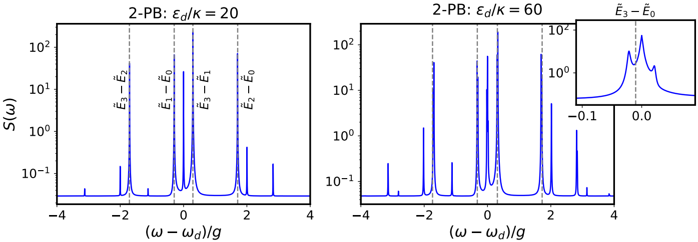
The intensity correlation function is

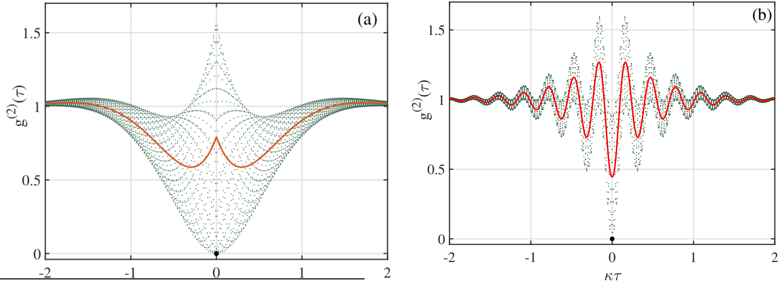
$$g^{(2)}(\tau) = \frac{{\rm tr}\left\{[e^{\mathcal{L}\tau}\rho_{\rm cond}]\sigma_+\sigma_-\right\}}{\langle\sigma_+\sigma_-\rangle_{\rm ss}} = 1 + e^{-\gamma|\tau|}[c_1\cos(2\Omega\tau) + c_2\sin(2\Omega|\tau|)$$
$$+ c_3 e^{-\gamma|\tau|} + c_4\cos(\nu\tau)], \quad \text{where} \quad c_i = f_i(p_3), \quad p_3 = \Omega^2/(4\Omega^2+\gamma^2). \tag{49}$$

For fluorescence, $g^{(2)}(\tau=0)=0$ always. For the transmitted field, $g^{(2)}_{\rightarrow}(\tau=0) \approx 1 + 3\gamma^2/(25\Omega^2)$ when $\Omega^2 \ll \gamma^2$ – extreme photon bunching[10].

[10] S. S. Shamailov et al., Opt. Commun. **283** (2010).

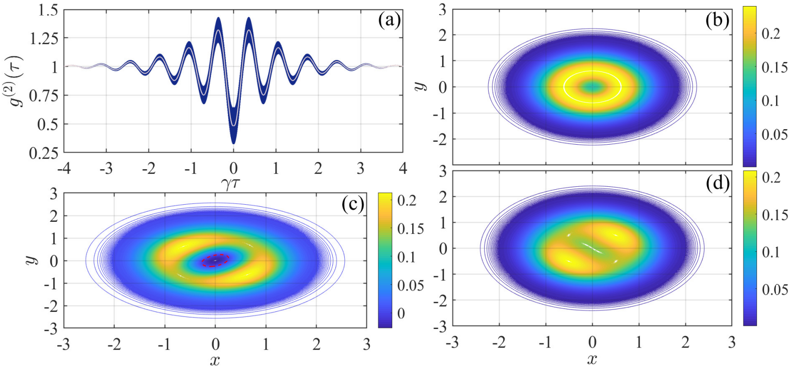
First and second-order coherence of fluorescence [11]





[11] Th. K. M. and C. Lledó, Optics Communications **486** (2021).

Second-order coherence and the Wigner function [12]



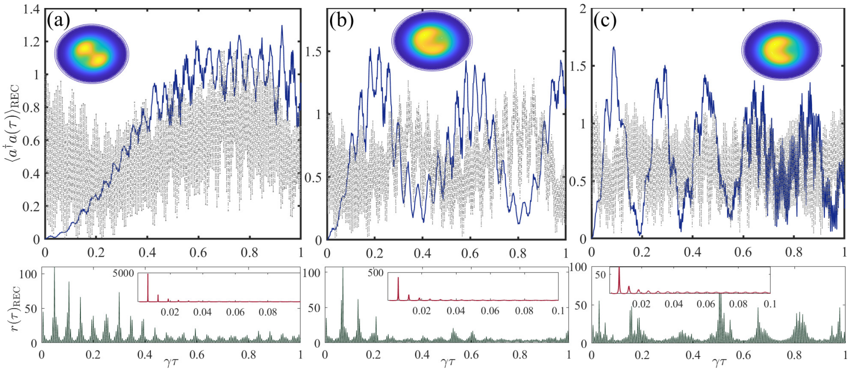
Transient and steady-state Wigner functions evince bimodality.

$$W_{\rm ss}(\alpha,\alpha^*) = \frac{2}{\pi} e^{-2|\alpha|^2} \Big\{ 4p_3|\alpha|^4 + 2p_3|\alpha|^2 + (1-3p_3) + i2\sqrt{p_3(1-4p_3)}\,[\alpha^2 - (\alpha^*)^2] \Big\}. \quad (50)$$

[12] Th. K. M., Phys. Rev. A **104** (2021).

Monitoring coherence in photon blockade V.

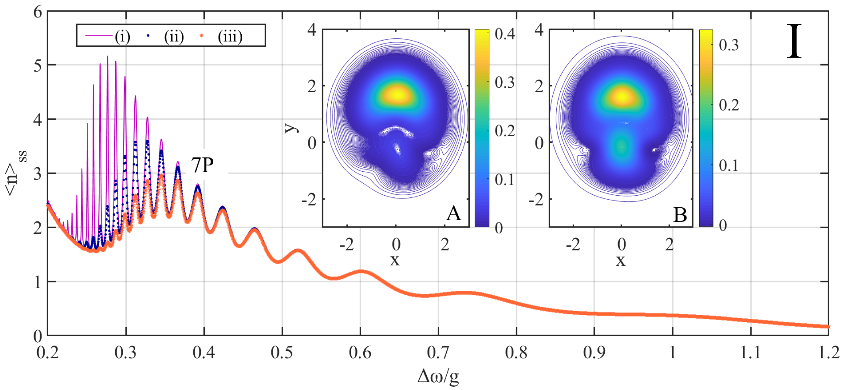
Resolving the conditional mixed state following spontaneous emission: $\rho_{\rm cond}(0) = \frac{2}{3}|0,-\rangle\langle 0,-| + \frac{1}{3}|1,-\rangle\langle 1,-|$. The 1/3 of realizations bring in prominently the quantum beat while the remaining 2/3 evince semiclassical oscillations. The conditional ratio of forwards to sideways emitted flux is modulated by the qunatum beat in bundles of peaks.



The transition saturates as we move from (a) to (c), and steady-state bimodality disappears.

The erasure of photon blockade

The two interactions (JC coupling and coupling to the coherent external drive) on equal footing: $|\varepsilon_d| \sim g/2$.

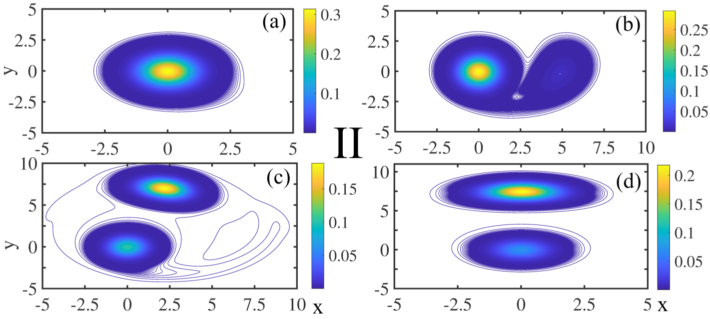


Resonances are 'erased'[13] for larger values of $|\varepsilon_d|/g$ or decreasing n_{sc}.

[13] Th. K. M, Phys. Rev. A **100** (2019).

Tracking a first-order dissipative phase transition

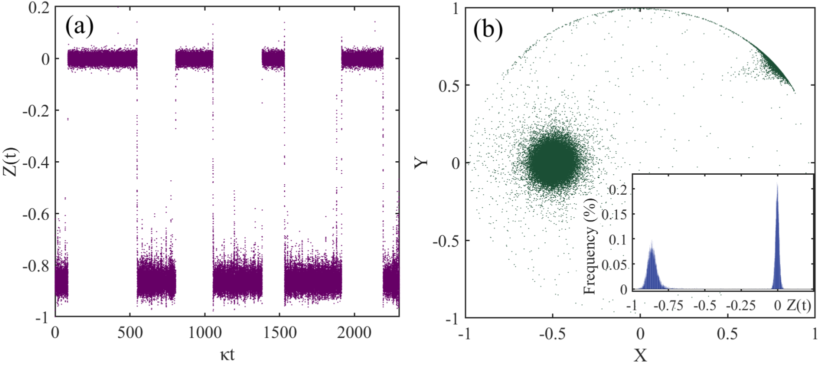
The two interactions on equal footing: $|\varepsilon_d| \sim g/2$.



- Now the "thermodynamic limit" restores a mean-field nonlinearity through complex-amplitude bistability.
- For strong excitation, $|\alpha_{\rm ss}|^2 = (|\varepsilon_d| \pm g/2)^2/(\Delta\omega)^2$. The "$-$" branch is unstable, heralding symmetry breaking at threshold.
- Compare with the weak-coupling limit of absorptive optical bistability, where $n_{\rm wc} = \gamma^2/(8g^2)$.

Two-level atom observables before symmetry breaking

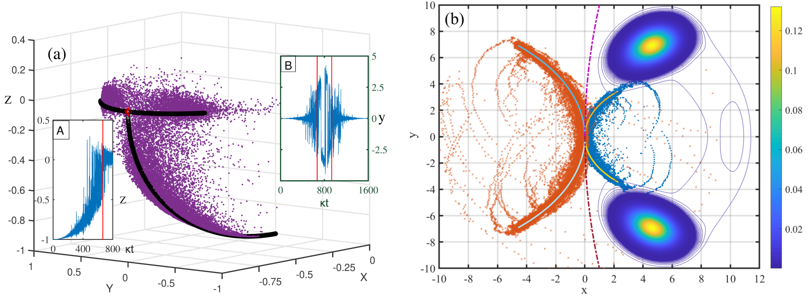
Inversion and polarization fluctuations close to resonant excitation:



- The two states have quasi-Poissonian statistics as shown by $\langle \sigma_z(t) \rangle$.
- The two distributions approach the equator of the Bloch sphere as $\omega_d \to \omega_A (= \omega_c)$ and $|\varepsilon_d| \to g/2$.

Symmetry breaking on resonance

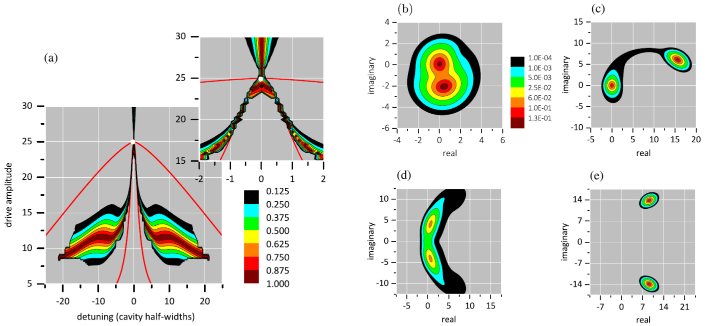
Spontaneous symmetry breaking for the two coupled degrees of freedom with growing n_{sc} (for $\Delta\omega = 0$)[14].



Quantum fluctuations stabilize the mean-field states as "attractors". A state that fluctuates between two complex-conjugate amplitudes heralds the mean-field bifurcation.

[14] Th. K. M, Phys. Rev. A **100** (2019).

- Critical point (white spot) of a **second-order** quantum dissipative phase transition in **zero dimensions**: $\gamma^2/(8g^2) \to 0$.
- Breakdown by means of amplitude and phase bimodality [15]:



- Coexistent states are revealed by the quantity $1 - |h_1 - h_2|/(h_1 + h_2)$, where $h_{1,2}$ are the peaks heights in the Q function.

[15] H. J. Carmichael, Phys. Rev. X **5**, 2015 (Fig. 2)

Concluding remarks

- Quantum systems of light-matter interaction out of equilibrium are subject either to a weak or a strong-coupling "thermodynamic limit".
- In the strong-coupling regime, the fluctuations have nothing in common with those envisaged by a **system size expansion**, where the outcomes of single quantum events are assumed microscopic, with only their cumulative effect appearing as a diffusion process at the microscopic level.
- *On resonance*, photon blockade breaks down by means of a dissipative first-order quantum phase transition where bistability sets in already for very low excitation. **Strong-coupling limit** with $n_{\rm sc} = [g/(2\kappa)]^2$ at which *fluctuations don't vanish*.
- *Photon correlations* in **mutltiphoton resonances** reveal the distinct $\sqrt{n}$ Jaynes-Cummings spectrum through a quantum beat (or a superposition of quantum beats), also leaving an imprint on single trajectories.
- The limit of "zero system size" ($n_{\rm wc} = [\gamma/(2\sqrt{2}g)]^2 \to 0$) on resonance accounts for new (semiclassical) stationary states predicting a threshold organizing a **second-order dissipative quantum phase transition**. Coherence is 'transferred' to a monitoring emitter in a cascaded systems setup.

Thank you for your attention!